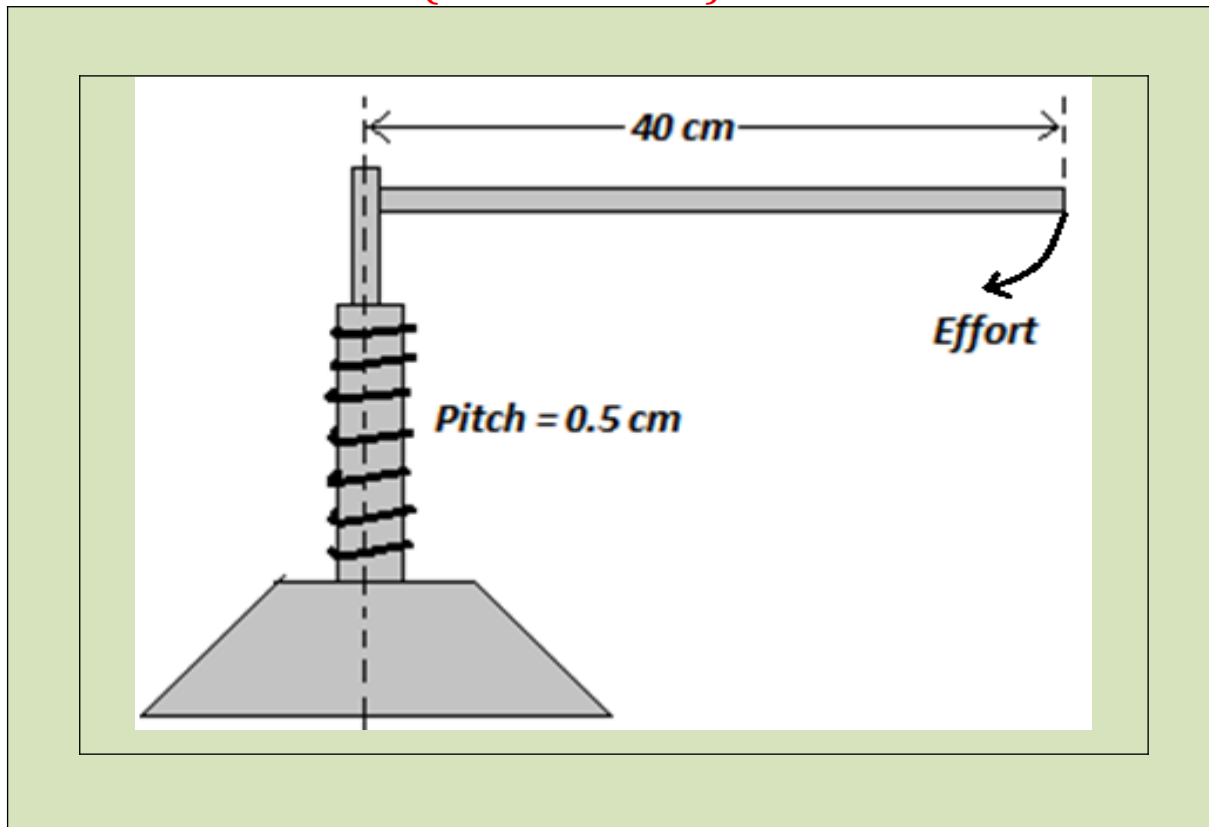


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FORM THREE PHYSICS **HANDBOOK**

[With well drawn diagrams, solved examples and questions for exercise]
(2015 Edition)



LABO ATOMS.

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I am very grateful to the entire Nyabururu Girls' High school fraternity for generously supporting me all round as I worked on this material. I must specifically appreciate the H.O.D Physics Nyabururu Girls' Mr. Albert O. Onditi for the support and encouragement.

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The care and best wishes I received from my mother Joyce Mokeira and my siblings deserve special attention. They were a great source of encouragement.

Lines that influence activities in my life

1. *God is always there to assist provided you ask for Him.*
2. *At its best, Physics eliminates complexity by revealing underlying simplicity.*
3. *There is no method of changing your fate except through hard work.*
4. *Cohesion with immediate neighbours and determination always betters your immediate environment.*

Brief Personal Profile (Cell Phone – 0726 593 003)

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Chapter One

LINEAR MOTION

<u>Specific objectives</u>	<u>Content</u>
<p>By the end of this topic the learner should be able to:</p> <ol style="list-style-type: none"> a) Define distance, displacement, speed, velocity and acceleration b) Describe experiments to determine velocity and acceleration c) Determine acceleration due to gravity d) Plot and explain motion- time graphs e) Apply the equation of uniformly accelerated motion f) Solve numerical questions. 	<ol style="list-style-type: none"> 1. Distance, displacement, speed, velocity, acceleration (experimental treatment required) 2. Acceleration due to gravity <ul style="list-style-type: none"> • free fall, • Simple pendulum method 3. Motion- time graphs <ul style="list-style-type: none"> • Displacement-time graphs • Velocity- time graphs 4. Equations of uniformly accelerated motion 5. Problems on uniformly accelerated motion

Introduction

- ❖ This topic deals with study motion of bodies in a straight line.

Terms Associated with Linear Motion

i. Distance

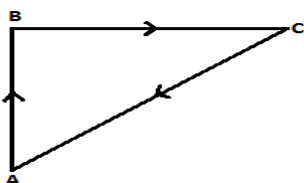
- ❖ Distance is the actual length covered by a moving body. It has no specific direction and it is therefore a **scalar quantity**. The SI unit of distance is **meter (m)**

ii. Displacement, s

- ❖ This is the distance covered by a moving body in a specified direction. Displacement is therefore a **vector quantity**. The SI unit of displacement is **metre (m)**

Illustrating distance and displacement

Consider the diagram below showing motion of a body starting from point A and moving in the direction shown.



- At point B, distance covered is AB while the displacement of the body is AB in the direction **AB**
- At point C, distance covered is AB + BC while the displacement is AC in the direction **AC**
- When back at starting point A, distance covered is AB + BC + CA while the displacement is **zero**.

iii. Speed

This is the rate of change of distance covered by a moving body with time. Speed is a **scalar quantity**.

$$\text{speed} = \frac{\text{distance covered}}{\text{time taken}}$$

For a body moving with a non-uniform speed,

$$\text{average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

- ❖ **Instantaneous speed** refers to the rate of change of distance of a moving body at a point (an instant). The SI unit of speed is **the metre per second (ms^{-1})**

iv. Velocity

- ❖ This is the rate of change of displacement with time. It can also be defined as the speed in a specified direction. Velocity is therefore a **vector quantity**.

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

For a body moving with a varying velocity,

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

- ❖ The SI unit of velocity is **the metre per second (ms^{-1})**.

v) Acceleration

- ❖ This is the change of velocity per unit time. It is a **vector quantity**.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

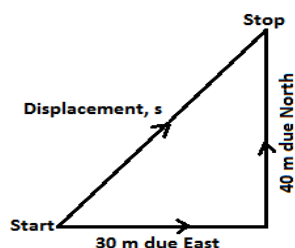
$$\text{Acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

<p>$a = \frac{v-u}{t}$, where, v is the final velocity, u is the initial velocity and t is the time taken.</p> <p>Instantaneous acceleration is the acceleration of a body at a point. Deceleration or retardation is the negative acceleration in which a body moves with a decreasing velocity with time. The SI unit of acceleration is metre per square second (ms^{-2}).</p>	<p>Solution</p> <p>$u = 2.0 \times 10^5 \text{ kmh}^{-1} = 5.55556 \times 10^4 \text{ ms}^{-1}$, $v = 0 \text{ ms}^{-1}$</p> <p>$a = \frac{v-u}{t}$; $a = \frac{(0 - 5.55556 \times 10^4) \text{ ms}^{-1}}{2.0 \times 10^{-2} \text{ s}} = -2.777778 \times 10^6 \text{ ms}^{-2}$</p> <p>Therefore retardation is $-(-2.777778 \times 10^6 \text{ ms}^{-2})$ $= 2.777778 \times 10^6 \text{ ms}^{-2}$</p>
<p>Examples</p>	<p>Exercise</p> <p>1) A van on a straight road moves with a speed of 180 kmh^{-1} for 45 minutes, and then climbs an escarpment with a speed of 72 kmh^{-1} for 30 minutes. Calculate:</p>

1. A body moves 30 m due east in 4 seconds, then 40 m due north in 8 seconds. Determine:

- The total distance moved by the body.
- The displacement of the body.
- The average speed of the body.
- The average velocity of the body.

Solution



- a) Total distance = total length covered

$$30 \text{ m} + 40 \text{ m} = 70 \text{ m}$$

- b) Total displacement is $(\sqrt{30^2 + 40^2}) \text{ m}$

$$\sqrt{2500} \text{ m} = 50 \text{ m on bearing } 36.87^\circ$$

- c) Average speed = $\frac{\text{total distance covered}}{\text{total time taken}}$
- $$= \frac{70}{12} = 5.833 \text{ ms}^{-1}$$

- d) Average velocity = $\frac{\text{total displacement}}{\text{total time taken}}$

$$= \frac{50}{12} = 4.167 \text{ ms}^{-1} \text{ on bearing } 36.87^\circ$$

I. The average speed of the van

II. The average acceleration produced

- 2) A girl runs 40 m due south in 40 seconds and then 20 m due north in 10 seconds. Calculate:

I. her average speed

II. her average velocity

III. her change in velocity for the whole journey

IV. The acceleration produced by the girl.

Measuring Speed, Velocity and Acceleration Using Ticker Timer

- ❖ A ticker timer has an arm which vibrates regularly due to changing current in the mains supply (alternating current). As the arm vibrates, it makes dots on the paper tape which is moving under the arm. **Successive dots are marked at the same interval of time**
- ❖ Most ticker timer operates at a frequency of 50 hertz (50Hz) i.e. 50 cycles per second i.e. they make 50 dots per second. The time interval between two consecutive dots is: $\frac{1}{50} = 0.02\text{s}$ for a 50Hz ticker timer. This time interval is called a **tick**.

Sample sections of tapes are as shown below. The arrow shows the direction in which the tape is pulled.

2. The speed of a body rolling on an inclined plane is 10 ms^{-1} when time is 0 s at time $t = 10 \text{ s}$ the speed of the body is found to be 25 ms^{-1} . If the body is moving in the same direction throughout, calculate the average acceleration of the body

Solution

$$a = \frac{v - u}{t}$$

$$a = \frac{(25 - 10) \text{ ms}^{-1}}{(10 - 0) \text{ s}}$$

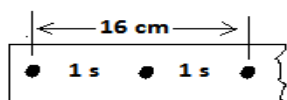
$$a = \frac{15 \text{ ms}^{-1}}{10 \text{ s}} = 1.5 \text{ ms}^{-2}$$

3. A particle moving with a velocity of $2.0 \times 10^6 \text{ kmh}^{-1}$ is brought to rest in $2.0 \times 10^2 \text{ s}$. calculate the acceleration of the body, hence the retardation.

Examples

1. A tape is pulled through a ticker timer which makes one dot every second. If it makes three dots and the distance between the first and the third dot is 16 cm , find the velocity of the tape.

Solution

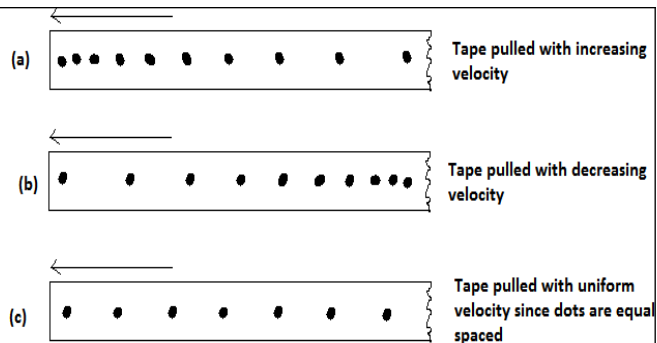


$$f = 1 \text{ Hz}$$

$$\text{Time between consecutive dots} = 1 \text{ s, total time} = 1 \text{ s} + 1 \text{ s} = 2 \text{ s}$$

$$\text{Distance between 1st and 3rd dots} = 16 \text{ cm} = 0.16 \text{ m}$$

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time taken}} = \frac{0.16 \text{ m}}{2 \text{ s}} = 0.08 \text{ ms}^{-1}$$



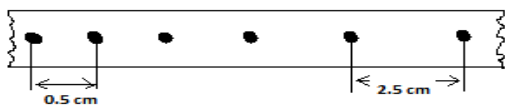
- I. Calculate the average velocity for the motion
- II. What does the area under the straight line represent?
- III. What is the difference between successive sections of tape?
- IV. Calculate the acceleration of the trolley in ms^{-1}

Motion Graphs

- ❖ Graphs can be used to represent variation of distance, speed, velocity or acceleration of a moving body with time. When used this way they are called **motion graphs**

Displacement – Time Graphs

3. The tape in the figure below was produced by a ticker timer with a frequency of 100Hz. Find the acceleration of the trolley that was pulling the tape. Solution



Solution

$$\text{time between consecutive dots} = \frac{1}{100} = 0.01 \text{ s}$$

$$\text{initial velocity, } v = \frac{0.005 \text{ m}}{0.01 \text{ s}} = 0.5 \text{ ms}^{-1}$$

$$\text{final velocity} = \frac{0.025 \text{ m}}{0.01 \text{ s}} = 2.5 \text{ ms}^{-1}$$

$$\text{time taken} = 4 \times 0.01 = 0.04 \text{ s}$$

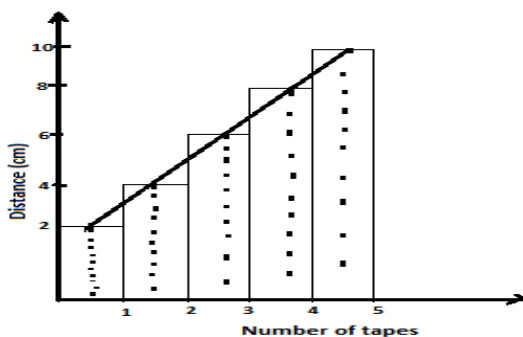
$$\text{acceleration} = \frac{v - u}{t} = \frac{(2.5 - 0.5) \text{ ms}^{-1}}{0.04 \text{ s}} = \frac{200}{0.04} = 50 \text{ ms}^{-2}$$

Exercise

1. The figure below shows a piece of tape pulled through a ticker timer by a trolley down an inclined plane. The frequency of the ticker timer is 50Hz

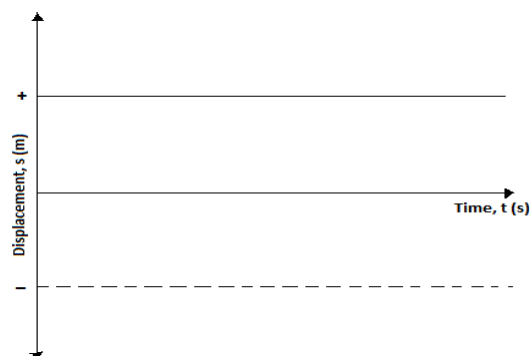
- I. What type of electric current is used to operate the ticker timer?
- II. Calculate the average velocity for the trolley between A and B

2. The figure below shows a tape chart from the paper tape obtained (frequency of ticker timer 50 Hz)



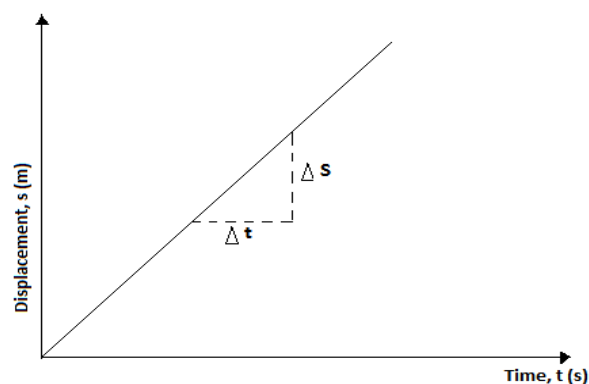
1. Stationary body

- ❖ Displacement does not change with time, since displacement is a vector quantity the position of the body may be negative or positive relative to be observer.



2. A body moving with uniform velocity

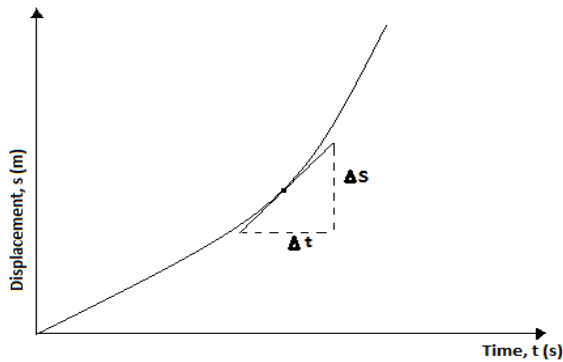
- ❖ For a body moving with uniform velocity, displacement changes uniformly over equal time intervals. The graph of displacement against time is a straight line whose slope or gradient represents the velocity of the body which is **constant**.



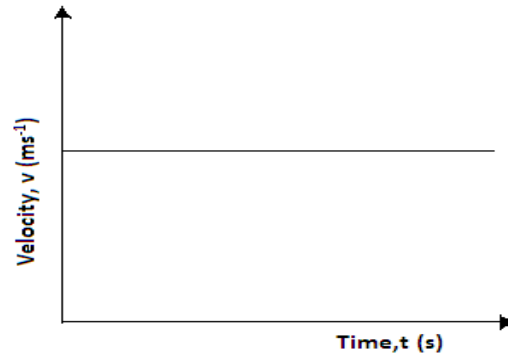
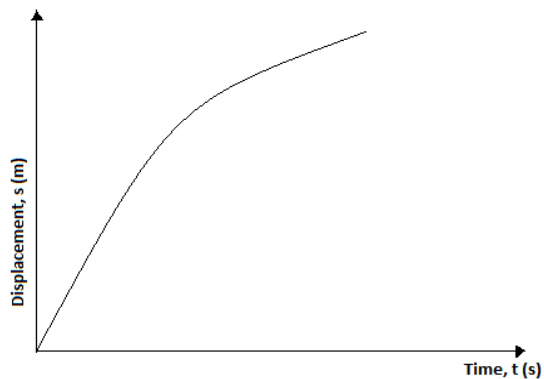
3. A body moving with variable velocity

- ❖ a. For a body moving with a **velocity increasing uniformly with time**, the displacement-time graph is a curve of increasing slope since the distance the body covers increases for equal time intervals. The slope of the graph at any given point gives instantaneous velocity of the body i.e.

$$\text{instantaneous velocity} = \frac{\Delta s}{\Delta t}$$

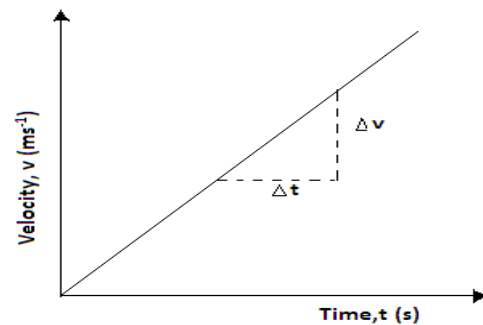


- ❖ b. For a body moving with a **velocity decreasing uniformly with time**, the displacement-time graph is curve of decreasing slope since the distance the body covers decreases for equal time intervals as shown below.



2. A body moving with its velocity changing uniformly

- ❖ The gradient of this graph is a straight line; meaning that velocity changes uniformly over equal time intervals. This gradient graph gives **constant acceleration** i.e. $\text{acceleration} = \frac{\Delta v}{\Delta t}$

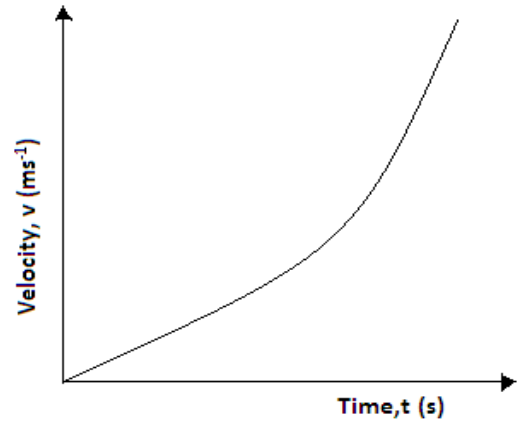


3. A body moving with its velocity changing non-uniformly

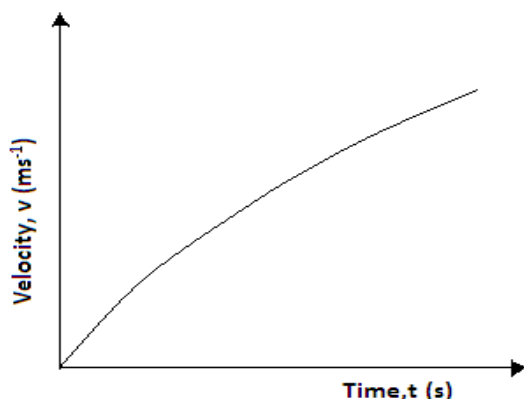
- ❖ a) For a body moving with an **increasing acceleration**, meaning that its velocity is increasing at an increasing rate, the velocity-time graph is curve of increasing slope as shown alongside. The **slope of the graph at any point gives the instantaneous acceleration** of the body at that point.

Velocity – Time Graphs**1. Body moving with uniform velocity**

- ❖ The slope/gradient of the graph is zero and therefore acceleration of the body moving with uniform velocity is zero.

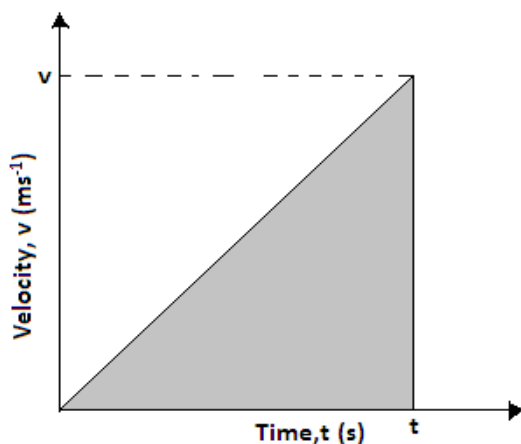


- ❖ b. For a body moving with a **decreasing acceleration**, meaning that its velocity is increasing at a decreasing rate, the velocity-time graph is curve of decreasing slope as shown below.



Area under velocity – time graph

- ❖ Consider a body starting from rest moving with constant acceleration for time, t . The velocity-time graph for the body is as shown alongside.



distance travelled = average velocity \times time

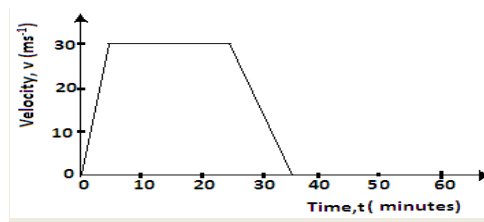
$$= \left(\frac{0 + v}{2} \right) \times t = \frac{1}{2}vt$$

- ❖ Therefore, **area A under the velocity – time graph represents the distance covered by the body** after time t seconds.

Solution

- *ab- the velocity of the car increases uniformly from rest (i.e. it accelerates uniformly)*
- *bc- the velocity of the car decreases uniformly to rest (i.e. it decelerates uniformly)*
- *cd- velocity of the car increases uniformly but in opposite direction (accelerates uniformly in opposite direction)*
- *de- velocity of the car decreases uniformly but still in same opposite direction (decelerates uniformly in opposite direction)*

2. A car starting from rest accelerates uniformly for 5 minutes to reach 30 ms^{-1} . It continues at this speed for the next 20 minutes and then decelerates uniformly to come to stop in 10 minutes. On the axes provided, sketch the graph of velocity against time for the motion of the car and hence, find the total distance covered by the car. Solution



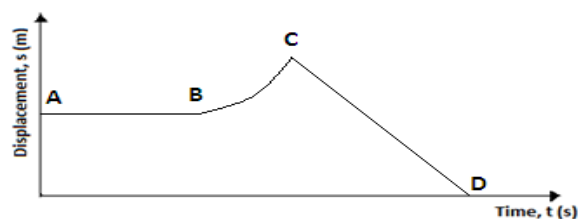
Total distance = total area under the graph

$$\text{Total distance} = \frac{1}{2}(35 + 20)60 \times 30 \text{ (i.e. area of a trapezium)}$$

$$\text{Total distance} = 49500 \text{ m}$$

Exercise

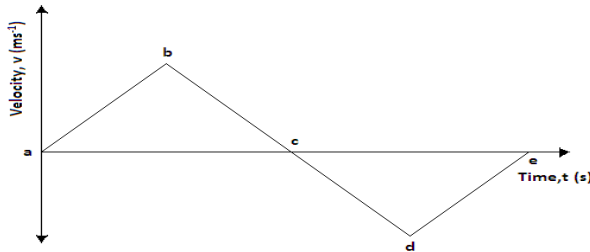
1. The figure below shows the displacement time graph of motion of a particle.



State the nature of the motion between: (i) A and B (ii) B

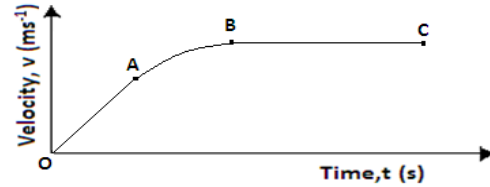
Examples

1. Interpret the graph below representing motion of a car from point a to e.

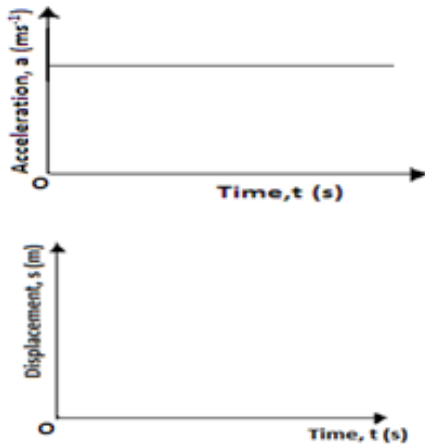


and C (iii) C and D

2. The figure below shows a velocity time graph for the motion of a certain body. Describe the motion of the body in the region: (i) OA (ii) AB (iii) BC.



3. The figure below shows the acceleration time graph for a certain motion. On the axes provided, sketch the displacement-time graph for the same motion.



Therefore, for a body moving with uniform acceleration, any of these three equations may be used.

$$1. v = u + at$$

$$2. s = ut + \frac{1}{2}at^2$$

$$3. v^2 = u^2 + 2as$$

Examples

1. A car starts from rest with uniform acceleration of 5ms^{-2} . How long does it take to cover a distance of 400m?

Solution

$$a = 5\text{ms}^{-2}$$

$$S = 400\text{m}$$

$$t = ?$$

$$U = 0\text{ms}^{-1}$$

Equations of Uniformly Accelerated Motion

- ❖ Consider a body moving in a straight line with uniform acceleration a , so that its velocity

increases from an initial value u to a final value v in time t and it is displaced by s ;

Derivation of the 1st equation

$$\text{acceleration, } a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v - u}{t}$$

$$at = v - u$$

$$v = u + at$$

Derivation of 2nd equation

displacement, s = average velocity \times time

$$s = \left(\frac{u + v}{2} \right) \times t$$

but from equation 1, $v = u + at$

$$s = \left(\frac{u + u + at}{2} \right) t$$

$$\frac{1}{2}(2ut + at^2)$$

$$s = ut + \frac{1}{2}at^2$$

Derivation of 3rd equation

displacement = average velocity \times time

$$s = \left(\frac{u + v}{2} \right) \times t$$

but from equation 1, $v = u + at$

$$t = \frac{v - u}{a}$$

$$s = \left(\frac{v + u}{2} \right) \times \left(\frac{v - u}{a} \right)$$

$$s = \frac{v^2 - uv + uv - u^2}{2a}$$

Best equation for use is; $s = ut + \frac{1}{2}at^2$

$$400 = 0 \times t + \frac{1}{2} \times 5 \times t^2$$

$$400 = \frac{5}{2}t^2$$

$$t = \sqrt{\frac{400 \times 2}{5}} = 12.65 \text{ seconds.}$$

2. A body is uniformly accelerated from rest to a final velocity of 100ms^{-1} in 10 seconds. Calculate the distance covered.

Solution

$$u = 0$$

$$v = 100\text{ms}^{-1}$$

$$t = 10\text{s}$$

$$s = ?$$

right equation is $v^2 = u^2 + 2as$

$$100^2 = 0^2 + 2as$$

$$\text{But } a = \frac{v - u}{t}$$

$$\frac{100 - 0}{10} = 10 \text{ ms}^{-2}$$

$$s = \frac{10000\text{m}^2\text{s}^{-2}}{2 \times 10} = 500 \text{ m}$$

3. A body whose initial velocity is 30ms^{-1} moves with a constant retardation of 3ms^{-2} . Calculate the time taken for the body to come to rest.

Solution

$$u = 30 \text{ ms}^{-1}$$

$$a = -3 \text{ ms}^{-2}$$

$$t = ?$$

$$v = 0\text{ms}^{-1}$$

Right equation is $v = u + at$

$$0 = 30 - 3t$$

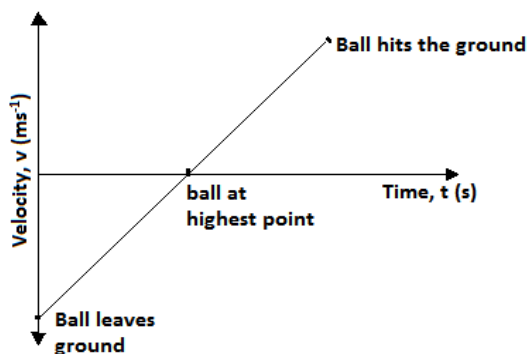
$2as = v^2 - u^2$ $v^2 = u^2 + 2as$	$-30 = -3t$ $t = 10s$ 4. A body moving with a uniform acceleration of $10ms^{-2}$ covers a distance of 320m. If its initial velocity was $60ms^{-1}$, calculate its final velocity. $a = 10ms^{-2}$ $s = 320m$ $u = 60ms^{-1}$ $v = ?$
Right equation is $v^2 = u^2 + 2as$ $v^2 = 60^2 + 2 \times 10 \times 320$ $v = \sqrt{3600 + 6400}$ $= \sqrt{10000}$ $= 100 ms^{-1}$	(b) Right equation is $v^2 = u^2 + 2gs$ $20^2 = 0^2 + 2 \times 10 \times s$ $s = \frac{400}{20} = 20m$

Motion under Gravity

Acceleration (In Free Fall) and Deceleration (In Vertical Projection) Due To Gravity

❖ When a body is projected vertically upwards, *it decelerates uniformly (negative acceleration) due to gravitational pull*. When this body falls from maximum height, it *accelerates uniformly* downwards and this is called *free fall*.

❖ Consider a ball thrown vertically upwards from the ground. The graph below shows how the velocity of the ball changes with time from when it leaves the ground until it hits the ground again. *Air resistance is assumed to be negligible*. Downward velocity is taken to be positive. The gradient of this graph is a constant whose value is *gravitational acceleration, g*.



❖ Equations of uniformly accelerated bodies also apply in motion under gravity.

$$1. v = u + gt$$

$$2. s = ut + \frac{1}{2}gt^2$$

2. A bullet shot vertically upwards rises to a maximum height of 1000m. Determine:

a) The initial velocity of the bullet

b) The time of flight of the bullet

Solution

(a) At maximum velocity, velocity is zero

$$v = 0 \text{ ms}^{-1}$$

$$u = ? \text{ ms}^{-1}$$

$$g = -10 \text{ ms}^{-1}$$

because the bullet decelerates upwards

$$s = 1000 \text{ m}$$

Using the equation $v^2 = u^2 + 2gs$

$$0 = u^2 + 2 \times (-10) \times 1000$$

$$0 = u^2 - 20000$$

$$u = \sqrt{20000} = 141.42 \text{ ms}^{-1}$$

(b) Total time the bullet is in air

$$\text{using the expression } s = ut + \frac{1}{2}gt^2,$$

displacement is 0 m

$$0 = 141.42 \times t + \frac{1}{2}(-10)t^2$$

$$141.42 = 5t$$

$$t = \frac{141.42}{5} = 28.28 \text{ s}$$

Exercise

$$3. v^2 = u^2 + 2gs$$

Examples

1. A stone is released vertically downwards from a high cliff. Determine

a) its velocity after two seconds

b) How far it has travelled after two seconds.

Solution

(a) $u = 0$

$t = 2\text{seconds}$

$g = 10\text{ms}^{-1}$

Right expression is

$v = u + gt$

$v = 0 + 10 \times 2$

$v = 20\text{ms}^{-1}$

1. A stone is released from a cliff of 180m high calculate

a) The time it takes to hit the ground

b) The velocity with which it hits the water
(take $g = 10\text{ms}^{-1}$)

2. A body is projected vertically upward with an initial velocity u . it returns to the same point of projection after 8s. Plot:

a) The speed time graph

b) The velocity time graph for the body

3. A body is thrown vertically upwards with an initial velocity of u . show that:

I. Time taken to reach maximum height is $t = \frac{u}{g}$

II. Time flight (time taken for the body(projectile) to fall back to point of projection) is $t = \frac{2u}{g}$

III. Maximum height reached is $H_{\max} = \frac{u^2}{2g}$

IV. Velocity of return is equal in magnitude to velocity of projection

4. A stone is projected vertically upward with a velocity of 30ms^{-1} from the ground. Calculate:

I. The time it takes to reach maximum height.

II. The time of flight.

III. The maximum height reached.

IV. The velocity with which it lands the ground.

Horizontal projection

Example

Some examples of horizontal projection include:

1. A jet from a water pipe held horizontally
2. A bullet from a gun held horizontally
3. A tennis ball when it rolls from the tennis table.
4. A stone thrown horizontally.
5. An arrow released horizontally from bow.

❖ Consider a body projected horizontally with an initial horizontal velocity **u**. The body maintains that initial horizontal velocity but since it also experiences free fall due to gravity, it describes a curved path as shown below.

1. ***An arrow is shot horizontally from the top of the building and it lands 200 m from the foot of the building after 10s. Assuming that the air resistance is negligible, calculate:***

a. The initial velocity of the arrow.

b. the height of the building

Solution

- a.** initial horizontal velocity, $u = ?$

$$R = 200\text{m}$$

$$t = 10\text{s}$$

From the expression $R = ut$;

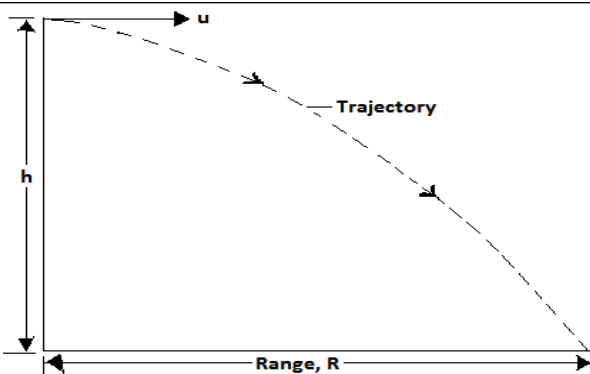
$$200\text{m} = u \times 10$$

$$u = \frac{200}{10} = 20\text{ms}^{-1}$$

- b.** from the expression; $h = \frac{1}{2}gt^2$

$$h = \frac{1}{2} \times 10 \times 10^2$$

$$= 5 \times 100 = 500\text{m}$$



- ❖ The path followed the body projected horizontally (projectile) is called **trajectory**. The maximum horizontal distance covered by the body projected horizontally is called the **range**. The **vertical acceleration is due to gravity** while the **horizontal acceleration is zero since the body maintains its initial horizontal velocity throughout the motion**.

- ❖ The displacement of the projectile at any given time t is given by $s = ut + \frac{1}{2}at^2$

Horizontal displacement, R

- ❖ Since horizontal acceleration, a , is zero, $R = ut$

The vertical displacement, h

- ❖ Initial velocity for vertical displacement is zero. This means that vertical displacement is;

$$s = 0 \times t + \frac{1}{2}gt^2$$

$$s = \frac{1}{2}gt^2$$

$$h = \frac{1}{2}gt^2$$

Note: The time for horizontal displacement is equal to time for vertical displacement at any given point.

Exercise

1. A ball is thrown from the top of a cliff 20m high with a horizontal velocity of 10ms^{-1} , calculate:
 - a. The time taken by the ball to strike the ground
 - b. The distance from the foot of the cliff to where the ball lands.
 - c. The vertical velocity at the time it strikes the ground.
2. A stone is thrown horizontally from the building that is 45m, high above a horizontal ground. It hits the ground at a point which is 60m from the foot of the building. Calculate the initial velocity of the stone.
3. A ball is thrown from the top of a cliff 20m high with a horizontal velocity of 10ms^{-1} , calculate:
 - I. The time taken by the ball to strike the ground
 - II. The distance from the foot of the cliff to where the ball lands.
 - III. The vertical velocity at the time it strikes the ground.
4. A stone is thrown horizontally for the building that is 45m, high above a horizontal ground. It hits the ground at a point which is 60m from the foot of the building. Calculate the initial velocity of the building.

NB: REMEMBER THE SIMPLE PENDULUM EXPERIMENT.

A pendulum is a small heavy body suspended by a light inextensible string.

Revision Exercise

1. A body moving at an initial velocity u (ms^{-1}) accelerates at a (ms^{-2}) for t seconds and attains a final velocity v (ms^{-1}). Represent this motion on the velocity against time axes shown below.

2. The data in the table below represents the motion over a period of 7 seconds

Time in s	0	1	2	3	4	5	6	7
D in m	0	20	40	60	80	95	105	110

- a) Plot on graph paper a graph of displacement (y -axis) against time.
- b) Describe the motion of the vehicle for the first 4 seconds.
- c) Determine the velocities at 4.5s and 6.5 s. Hence or otherwise determine the average acceleration of the vehicle over this time interval.
3. A ball-bearing X is dropped vertically downwards, from the edge of a table and it takes 0.5s to hit the floor below. Another bearing Y leaves the edge of the table horizontally with a velocity of 5m/s. find:
- The time taken for bearing Y to reach the floor.
 - The horizontal distance traveled by Y before hitting the floor.
 - The height of the table-top above the floor level.
4. A helicopter, which was ascending vertically at a steady velocity of 20m/s, released a parcel that took 20second to reach the ground.

- State the direction in which the parcel moved immediately it was released.
- Calculate the time taken by the parcel to reach the ground from the maximum height.
- Calculate the velocity of the parcel when it strikes the ground.
- Calculate the maximum height above the ground the parcel reached.
- What was the height of the helicopter at the instant the parcel was dropped.

5. A stone is thrown horizontally from a building that is 50 m high above a horizontal ground. The stone hits the ground at a point, which is 65m from the foot of the building. Calculate the initial of the stone.

6. The figure represents dots made by a ticker-timer. The dots were made at a frequency of 50 dots per second. (Diagram not drawn to scale)

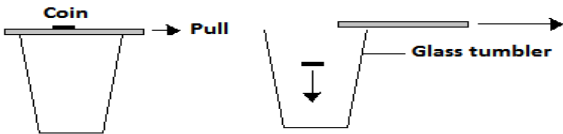
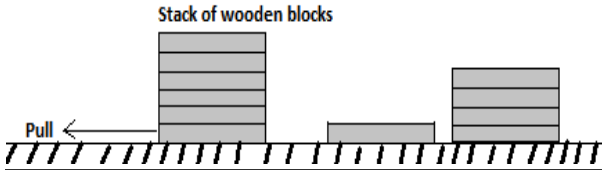


- What is time interval between two consecutive dots?
- The first dot from the left was made at time $t = 0$. Copy the diagram and indicate using arrows pointing downwards the dots made at $t = 0.1\text{s}$, 0.2s , 0.3s .
- Determine the average velocities of the tape over time intervals -0.02s to 0.02s , 0.08s to 0.12s , 0.18s to 0.22s and 0.28s to 0.32s
- Draw a suitable graph and from it determine the acceleration of the tape.

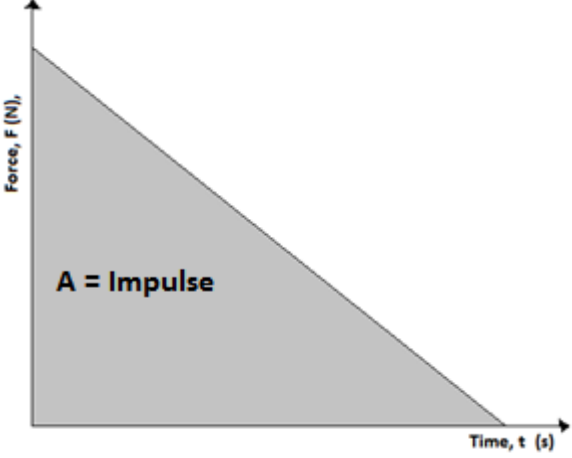
Chapter Two

NEWTON'S LAWS OF MOTION

<u>Specific Objectives</u>	<u>Content</u>
<p>By the end of this topic, the learner should be able to:</p> <ol style="list-style-type: none"> State Newton's laws of motion Describe simple experiments to illustrate inertia State the law of conservation of linear momentum Define elastic collision, inelastic collision and impulse Derive the equation $F=ma$ Describe the application of frictional force Define viscosity Explain terminal velocity Solve numerical problems involving Newton's laws and the law of conservation of linear momentum. 	<ol style="list-style-type: none"> Newton's laws of motion (experimental treatment of inertia required) Conservation of linear momentum: elastic collision, inelastic collisions, recoil velocity, impulse (oblique collisions not required) $F=ma$ Frictional force <ul style="list-style-type: none"> Advantages and disadvantages Viscosity Terminal velocity (qualitative treatment) Problem on Newton's law of conservation of linear momentum (exclude problems on elastic collisions)

<p>Introduction</p> <ul style="list-style-type: none"> ❖ Effects of force on motion of a body are based on Newton's three laws motion. In this topic, these laws are looked into. <p>Newton's First Law (The Law of Inertia)</p> <ul style="list-style-type: none"> ❖ It states that <i>"a body remains in its state of rest or of uniform motion in a straight line unless acted upon by an external force"</i>. This law is also called the <i>law of inertia</i> <p>Definition of Inertia</p> <ul style="list-style-type: none"> ❖ Is the tendency of a body to remain in its state of rest if it was at rest or in its state of motion if it was in motion. <p>Examples of Inertia (i.e. Examples of Newton's First Law in Practice)</p> <ol style="list-style-type: none"> 1. When a card supporting a coin on a glass tumbler is suddenly flicked, it is observed that the card flies off but the coin falls in the tumbler. This is because the coin tends to maintain its state of rest (it falls in glass because another force acts on it. Which is that other force?) 	<ol style="list-style-type: none"> 2. The bottom wooden block can easily be pulled out of the stack without disturbing others placed on it. The other blocks remain in a pile undisturbed except that their position is lowered because of downward pull of the gravitational force.  <ol style="list-style-type: none"> 3. When a moving train or a car stops suddenly, passengers are thrown forward. The passenger in a moving vehicle is also in a state of motion. Hence, when the vehicle stops suddenly, the upper part of his body continues to move. <i>This is why it is necessary to wear seat belt.</i> 4. A cyclist on a level ground continues to move for some time without pedaling. <p>Definition of Force as per Newton's First Law of Motion</p> <ul style="list-style-type: none"> ❖ Force is defined as that which produces motion in body at rest or which alters its existing state of motion.
<p>Momentum</p> <ul style="list-style-type: none"> ❖ Momentum of a body is defined as <i>the product of its mass and velocity</i>. For a body of mass m in kg and velocity v in ms^{-1}, $\text{momentum} = \text{mass}(\text{kg}) \times \text{velocity}(\text{ms}^{-1})$ $P = mv$	<p>Relationship between Force, Mass and Acceleration</p> <p>Consider a force F acting on a body of mass m for a time t. if its velocity changes from u to v, then;</p> <p>change in momentum = final momentum - initial momentum</p> $\Delta p = mv - mu$

<p>❖ SI unit of momentum is the kilogram metre per second (kgms^{-1}). Momentum is a vector quantity since it has both magnitude and direction. The direction of momentum is same as that of the velocity of the body.</p>	<p>Rate of change of momentum = $\frac{\text{change in momentum}}{\text{time}}$;</p> $\frac{\Delta p}{t} = \frac{mv - mu}{t}$ <p>From Newton's second law, $F \propto \frac{mv - mu}{t}$</p> $F \propto m\left(\frac{v - u}{t}\right); \text{ but } \frac{v - u}{t} = a$
Examples	
<p>1. What is the momentum of a racing car of mass 500kg driven at 270km/h?</p> <p>Solution</p> <p>$v = 270 \text{ kmh}^{-1} = 75 \text{ ms}^{-1}$; $m = 500 \text{ kg}$</p> <p>momentum = mass(kg) \times velocity(ms^{-1})</p> <p>momentum = $500 \text{ kg} \times 75 \text{ ms}^{-1}$</p> <p style="text-align: center;">$= 37500 \text{ kgms}^{-1}$</p>	<p>therefore, $F \propto ma$</p> <p>so, $F = kma$. where k is a constant of proportionality.</p> <p>When $F = 1\text{N}$, $a = 1\text{ms}^{-2}$, $m = 1\text{kg}$, then. $k = 1$</p> <p style="text-align: center;">hence, $F = ma$,</p> <p style="text-align: center;">This is an expression for Newton's second law</p> <p>Definition of a newton as per Newton's 2nd law of motion</p> <p>❖ A newton is the force which produces an acceleration of 1ms^{-2} when it acts on a body of mass 1kg.</p>
<p>2. Find the momentum of:</p> <p>a. An object of a mass 100g moving at 20ms^{-1}</p> <p>Solution</p> <p>momentum = mass(kg) \times velocity(ms^{-1})</p> <p style="text-align: center;">$= 0.100 \text{ kg} \times 20 \text{ ms}^{-1}$</p> <p style="text-align: center;">$= 2 \text{ kgms}^{-1}$</p> <p>b. An object of mass 2.0kg which falls from rest for 10s (Momentum after 10s).</p> <p>Solution</p> <p>$u = 0 \text{ ms}^{-1}$, $t = 10 \text{ s}$, $g = 10 \text{ ms}^{-2}$, $v = ?$</p> <p>using the expression $v = u + gt$,</p> <p style="text-align: center;">$v = 0 + 10 \times 10$</p> <p style="text-align: center;">$= 100 \text{ ms}^{-1}$</p> <p>$\text{mom} = 0.1 \times 100 = 100 \text{ kgms}^{-1}$</p>	<p style="text-align: center;">Examples</p> <p>1. What force is needed to stop a 500kg car moving at 180kmh^{-1} in 12.5m?</p> <p>Solution</p> <p>$m = 500 \text{ kg}$, $v = 0 \text{ ms}^{-1}$,</p> <p>$u = 180 \text{ kmh}^{-1} = 50 \text{ ms}^{-1}$, $s = 12.5 \text{ m}$, $F = ?$</p> <p>From the equation, $v^2 = u^2 + 2as$, $a = \frac{v^2 - u^2}{2s}$</p> <p>$a = \frac{0^2 - 50^2}{2 \times 12.5}$,</p> <p>$a = -100 \text{ ms}^{-2}$</p>
Exercise	

<p>1. What is the momentum of a racing car of mass 500kg driven at 270km/h?</p> <p>2. An apple of mass 100g falls a distance of 2.5m to the ground from a branch of a tree.</p> <p>a) Calculate the speed at which it hits the ground and the time taken for it to fall. (Ignore air resistance).</p> <p>b) Calculate the momentum of the apple just before hitting the ground</p>	<p>From Newton's 2nd law, $F = ma$, $F = 500 \times -100$</p> <p>$F = -50\,000\text{ N}$.</p> <p>what does the negative mean?</p> <p>2. An external force applied to a ball of mass 160g increases its velocity from 2.5cms^{-1} to 275cms^{-1} in 10 seconds. Calculate the force applied.</p>
<p>Newton's Second Law of Motion</p> <p>❖ It states that <i>"the rate of change of momentum of a body is directly proportional to the resultant external force producing the change and takes place in the direction of the force"</i>.</p> <p>Resultant force acting \propto rate of change of momentum</p>	<p>Solution</p> <p>$m = 160\text{ g} = 0.160\text{kg}$, $u = 2.5\text{ cms}^{-1} = 0.025\text{ ms}^{-1}$,</p> <p>$v = 275\text{ cms}^{-1} = 2.75\text{ ms}^{-1}$, $F = ?\text{N}$</p> <p>$F = ma$, $F = m\left(\frac{v-u}{t}\right)$</p> <p>$F = 0.160\left(\frac{2.75 - 0.025}{10}\right) = 0.0436\text{ N}$</p>
<p>Exercise</p>	 <p>❖ Note: Impulse occurs when bodies collide and the impulsive force is the one which causes destruction during collision. The time for which this impulsive force acts determines the extent of damage caused. If time of impact is long, damage is less than when time of impact is short. The following are some examples of designs made to prolong time of impact and therefore reduce damage by impulsive force.</p>
<p>1. What is the mass of an object which is accelerated at 5 ms^{-1} by a force of 200 N?</p> <p>2. A gun fires a bullet of mass 10.0g horizontal at 50 ms^{-1} at a fixed target of soft wood. The bullet penetrates 50 cm into a target. Calculate</p> <p>a) Time taken by the bullet to come to rest in the wood</p> <p>b) The average retarding force exerted by the wood on the bullet.</p> <p>3. A trolley of mass 1.5kg is pulled along by an elastic cord given an acceleration of 2ms^{-2}. Find the frictional force acting on the trolley if the tension in the cord is 5N.</p> <p>4. State Newton's second law of motion. Hence, show that $F = ma$.</p> <p>5. Define the newton (unit of force)</p> <p>6. A car of mass 1500kg is brought to rest from a velocity of 25ms^{-1} by a constant force of 3000N. Determine the change in momentum produced by the force and the time</p>	

that it takes to come to rest.

7. A hammer of mass 800 g produces a force of 400 N when it strikes the head of a nail. Describe how it is possible for the hammer to drive the nail into a piece of wood, yet a weight of 400 N resting on the head of the nail would not
8. A resultant force F acts on a body of mass m causing an acceleration a_1 on the body. When the same force acts on a body of mass $2m$, it causes an acceleration a_2 . Express a_2 in terms of a_1 .

1. Eggs are packed in spongy crates
2. Smart phones are put in soft holders
3. vehicles are fitted with safety airbags
4. some vehicles have collapsible bumpers and steering
5. High jumpers usually land on soft ground etc.

Impulse

- ❖ **Impulse** is defined as the product of force acting on a body and the time in which the force acts. **Impulsive force** refers to the force which acts on a body for a very short time during a collision.
- ❖ If a force F acts on a body of mass m for time, t , then the impulse of the force is given by:

$$\text{Impulse} = \text{force} \times \text{time}$$

$$\text{Impulse} = Ft$$

(SI unit of impulse is the newton second (Ns))

$$\text{But from Newton's second law of motion, } F = \frac{mv - mu}{t}$$

$$Ft = mv - mu, \text{ but } Ft = \text{impulse and } mv - mu$$

is the change in momentum of the body

- ❖ This implies that **the impulse of force acting on a**

Examples

1. Determine the change in momentum produced when a force of 4000 N acts on a body which is at rest for 0.003 minutes

Solution

$$F = 4000 \text{ N, time, } t = 0.003 \text{ minutes} = 0.18 \text{ s}$$

$$\text{Impulse} = \text{change in momentum} = Ft$$

$$\begin{aligned} \text{change in momentum} &= 4000 \times 0.18 \\ &= 720 \text{ kgms}^{-1} \text{ or } 720 \text{ Ns} \end{aligned}$$

2. A car of mass 400 kg starts from rest on a horizontal track. Find the speed 4 s after starting if the tractive force by the engine is 500 N.

Solution

$$\text{Impulse} = \text{change in momentum, } Ft = m(v - u)$$

body during some time interval is equal to the change in momentum produced in that body in that time.

- ❖ The **area under the plot of force F against time (t)** represents **impulse** or **change in momentum** during a collision.

$$500 \times 4 = 400(v - 0)$$

$$v = \frac{500 \times 4}{400} = 5 \text{ ms}^{-1}$$

Exercise

1. An apple of mass 100g falls a distance of 2.5m to the ground from a branch of a tree.

i. Calculate the speed at which it hits the ground and the time taken for it to fall. (Ignore air resistance).

- II. Assuming the apple takes 100 milliseconds to come to rest Calculate the average force experienced by the apple.

2. The table below shows the values of the resultant force, F , and the time t for a bullet traveling inside the gun barrel after the trigger is pulled.

Force, F (N)	360	340	300	240	170	110
Time, (t) (millisecond s)	3	4	8	12	17	22

i. Plot a graph of Force, F , against time t .

ii. Determine from the graph:

a) The time required for the bullet to travel the length of the barrel assuming that the force becomes zero just at the end of the barrel.

b) The impulse of the force.

c) Given that the bullet emerges from the muzzle of the gun with a velocity of 200 m/s, calculate the mass of the bullet.

3. A body of mass 5 kg is ejected vertically from the ground when a force of 600N acts on it for 0.1s. Calculate the velocity with which the body leaves the ground.

4. A high jumper usually lands on a thick soft mattress. Explain how the mattress helps in reducing the force of impact.

C. Lift moving downwards with acceleration

- ❖ The downward acceleration is negative and this is why the one feels lighter when lift is accelerating downwards. Therefore reading on the machine (apparent weight of the body in lift) is:

$$P' = mg - ma$$

Notes:

i. If the lift moves with constant velocity, the machine will read weight of the body since acceleration will be zero.

ii. If $a = g$ (free fall) the body will experience weightlessness since the reaction from the lift on the body will be zero.

Exercise

1. A lady of mass 80 kg stands on weighing machine in a lift. Determine the reading on the weighing machine when the lift moves:

a) downwards at a constant velocity of 2.0 ms^{-1}

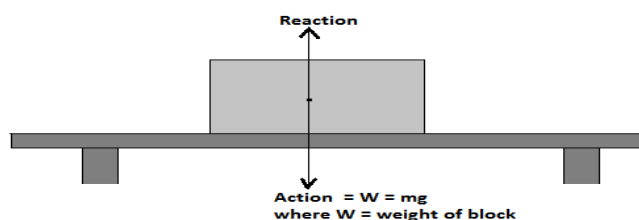
b) downwards with an acceleration of 3 ms^{-2}

c) upwards with an acceleration of 3 ms^{-2}

2. A man of mass 80 kg stands on a lift which is accelerating upwards at 0.5 ms^{-2} . if $g = 10 \text{ N/kg}$ determine the reaction on the man by the floor of the

Newton's Third Law of Motion

It states that ***"for every action, there is an equal but opposite reaction force"***. This means that if a body P exerts a force on another body Q, Q exerts an equal and opposite force on P. *It is clear that it is due to action (force exerted by foot on ground) and reaction (force exerted by earth on foot) that we are able to walk forward.*



Weight in a lift

a. Lift at rest

- ❖ Lift machine reads the weight of the body in lift since action and reaction are equal and opposite i.e. $R = mg$.

b. Lift moving upwards with acceleration a

The resultant upward force F produces the acceleration ($F = ma$)

resultant upward force F

= total upward force P - downward force (weight) W

$$F = P - W \Rightarrow P = F + w \text{ and } \therefore$$

$$P = mg + ma$$

- ❖ This is what the lift machine will read (the reaction of the lift on the body).

lift.

The Law of Conservation of Linear Momentum

- ❖ This law states that, ***"for a system of colliding bodies, their total linear momentum is a constant, provided no external forces are acting"***.
- ❖ If a body A of mass m_A initially moving with a velocity of u_A collides linearly with a body B of mass m_B initially moving with a velocity u_B and their velocities after collision are v_A and v_B respectively, then:

Total linear momentum before impact

= Total linear momentum after collision

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

Note: velocity is a vector quantity and must be

treated appropriately in calculations

Types of Collisions

1. Elastic Collision

- ❖ This is ***a collision in which both kinetic energy and momentum are conserved***. If the bodies A and B above collide elastically;

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$\frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

- ❖ **Note: In this collision, bodies *separate* and *move in same or different directions* after collision.**

II. Inelastic Collision

❖ This is **a collision in which momentum is conserved but kinetic energy is not**. In this collision, colliding bodies **fuse and move together** in one direction with a **common velocity**.

❖ If the collision of bodies A and B above collide inelastically, then;

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$\frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 \neq \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

❖ **Note:** the total kinetic energy after the impact is always less than the total kinetic energy before the impact and the loss is due to:

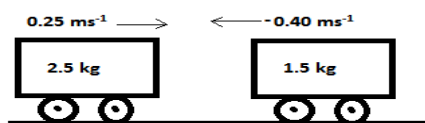
- I. Energy used in deformation of bodies
- II. Energy transformed to heat, sound and even light

Examples

1. Two trolleys of masses 2 kg and 1.5 kg are traveling towards each other at 0.25 m/s and 0.40 m/s respectively. The trolleys combine on collision.

- I. Calculate the velocity of the combined trolleys.
- II. In what direction do the trolleys move after collision?

Solution



Total linear momentum before impact
= Total linear momentum after collision

$$m_A u_A + m_B u_B = (m_A + m_B) v$$

$$m_b u_b + m_w u_w = (m_b + m_w) v$$

$$0.020 \times 50 + 1.980 \times 0 = (0.020 + 1.980) v$$

$$v = \frac{1}{2} = 0.5 \text{ ms}^{-1}$$

II. at maximum height, all K.E is converted to P.E

$$\frac{1}{2} m v^2 = m g h, \quad m \text{ cancels out}$$

$$\frac{1}{2} \times 0.5^2 = 10 \times h$$

$$h = \frac{0.125}{10} = 0.0125 \text{ m}$$

Exercise

1. A lorry of mass 3000 kg travelling at a constant velocity of 72 kmh⁻¹ collides with a stationary car of mass 600 kg. The impact takes 1.5 seconds before the two move together at a constant velocity for 15 seconds. Calculate:

- i. The common velocity
- ii. The distance moved after the impact
- iii. The impulsive force
- iv. The change in kinetic energy

2. A bullet of mass 15 g is shot from a gun 15 kg with a muzzle velocity of 200 ms⁻¹. If the bullet is 20 cm long, calculate:

- I. the acceleration of the bullet
- II. the recoil velocity of the gun

3. Explain how:

- i. Rocket propulsion takes place
- ii. Garden sprinkler works

$$\Rightarrow 2.5 \times 0.25 + 1.5 \times (-0.40) = (2.5 + 1.5)v$$

$$v = \frac{0.25}{4.0} = 0.0625 \text{ ms}^{-1}$$

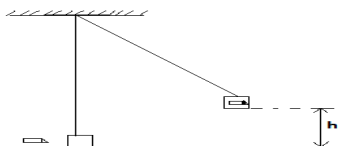
- III. They move in the direction to which trolley of mass 2.5 kg was moving to before collision

2. A bullet of mass 20 g travelling horizontally at a speed of 50 ms^{-1} embeds itself in a block of wood of mass 1980 g suspended from a light inextensible string. Find:

- I. The velocity of the bullet and block immediately after collision

- II. The height through which the block rises

Solution



Note. Consider a block of wood being pulled using a

Frictional Force

- ❖ Frictional force refers to the force that opposes or tends to oppose relative motion between two surfaces in contact. Frictional force acts in the direction opposite to that of the pulling force.

Types of Frictional Force

I. Static/ Limiting Frictional Force

- ❖ This is the force required to just start the body sliding. The force is directly proportional to the reaction force on the body by the surface.

$$F_s = \mu_s R,$$

where μ_s is the coefficient of static or limiting

frictional force

II. Kinetic/ Sliding Frictional Force

- ❖ This is the force required to keep the body sliding or moving at a constant speed. It opposes motion between two surfaces that are in relative motion. Sliding friction is directly proportional reaction force.

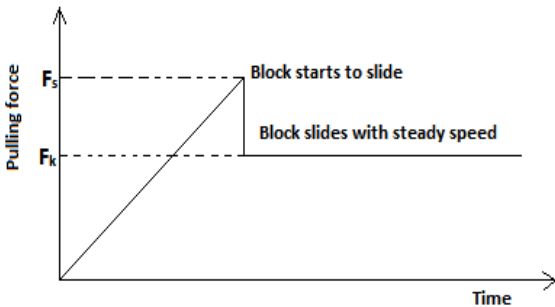
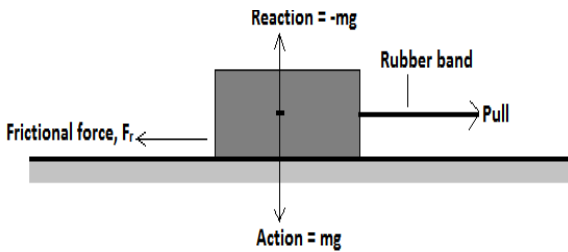
$$F_k = \mu_k R,$$

where μ_k is the coefficient of kinetic or

sliding frictional force

Exercise

rubber band on a horizontal surface as shown below.



- ❖ The rubber band stretches for some time before the block starts moving (stretch is due to limiting frictional). The stretching increases to a point when the block starts sliding steadily (sliding frictional force limits motion at this point). $\mu_s > \mu_k$
- ❖ **Frictional force is useful/ advantageous** in walking, moving vehicles, braking, writing, lighting a match stick etc.
- ❖ **Frictional force can be disadvantageous** as it causes wear and tear in moving parts of machine and leads to generation of unnecessary noise.

Example

1. A bullet of mass 10g traveling horizontally with a velocity of 300m/s strikes a block of wood of mass 290g which rests on rough horizontal floor. After impact they move together and come to rest after traveling a distance of 15m.
 - I. Calculate the common velocity of the bullet and the block.
 - II. Calculate the acceleration of the bullet and the block.
 - III. Calculate the coefficient of sliding friction between the block and the floor.
2. Under a driving force of 4000N, a car of mass 1250 kg has an acceleration of 2.5 m/s^2 . Find the frictional force acting on the car.
3. A bullet of mass 22g travelling with a horizontal velocity of 300 ms^{-1} strikes a block of wood of mass 378g which rests on a rough horizontal surface. After the impact, the bullet and the block move together and come to rest when the block has travelled a distance of 5m. Calculate the coefficient of sliding friction between the block and the floor.
4. A block of a metal A having a mass of 40kg requires a horizontal force of 100N drag it with uniform velocity a long horizontal surface.
 - I. Calculate the coefficient of friction
 - II. Determine the force required to drag a similar block having a mass of 30kg along the same horizontal surface, calculate.
 - III. If the two blocks A and B are connected with a tow bar and a force of 200N is applied to pull the two long the same surface, calculate.
 - a) The tension in the tow bar
 - b) The acceleration

A smooth wooden block is placed on a smooth wooden table. A force of 14N should be exerted on the 4kg wooden block to keep the block moving with a constant velocity.

a) What is the coefficient of kinetic friction, μ_k

Solution

$$F_k = \mu_k R, \quad F_k = \mu_k mg$$

$$14 = \mu_k \times 4 \times 10$$

$$\mu_k = \frac{14}{40} = 0.35$$

b) If a 20N brick is placed on the block, what force will be required to keep the block and brick moving with required constant speed?

Solution

coefficient of kinetic friction, μ_k remains constant

$$F_k = \mu_k R \Rightarrow F_k = 0.35 \times (40 + 20)$$

$$= 20 \text{ N}$$

c) **Use of ball bearing** - This is applied on rotating axles. The bearing allows the movement of the surface over the other.

d) **Air cushioning** - This is done by blowing air into the space between surfaces. This prevents surfaces from coming into contact since air is matter and occupies space.

Viscosity

- ❖ This is the force that opposes relative motion between layers of the fluid.
- ❖ Consider a small ball bearing introduced gently into glycerine in a long cylindrical jar. The forces acting on the ball are as shown below.

IV. If the tow bar is removed and the 40kg blocks of metal moves around a smooth path of radius 10m at a constant speed of 24ms⁻¹ calculate the centripetal force.

V. At the end of the circular path, the 40kg mass drops vertically in a trench 10m high and falls freely determine the time it takes to land at the bottom of the trench.

Factors Affecting Frictional Force between Two Surfaces in Contact

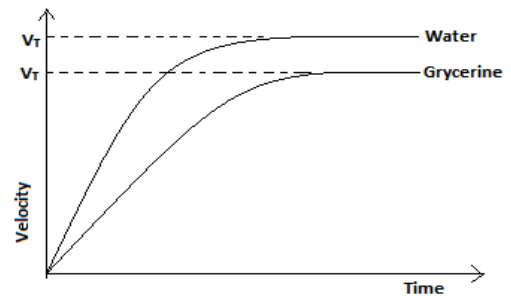
- I. **Normal reaction** - frictional force is directly proportional to normal reaction.
- II. **Nature of the surfaces** - the more rough a surface is, the larger the frictional force.

Methods of minimizing friction

- a) **Using rollers** - The rollers are laid down on the surface and the object pushed over them
- b) **Lubrication** - Application of oil or grease to the moving parts

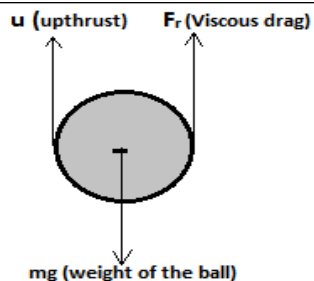
Factors Affecting Viscosity In Fluids

- I. **Density** - The higher the density of the fluid, the greater the viscous drag and therefore the lower the terminal velocity.



- II. **Temperature** - In liquids, viscous drag decreases (terminal velocity increases) with temperature, while in gases viscous drag increases (terminal velocity decreases) with temperature.

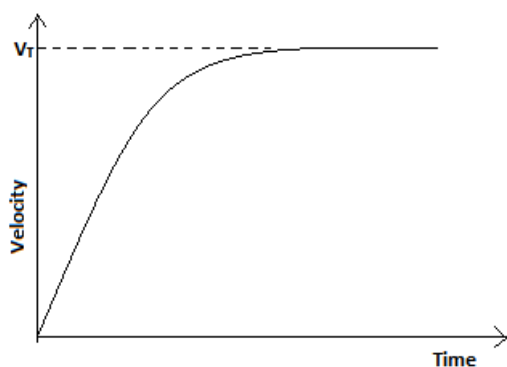
Exercise



- ❖ The resultant downward force ($mg - (u + F_r)$) accelerates the ball downwards.
- ❖ The viscous drag increases with velocity until the sum of upward forces equal the downward forces,

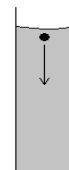
$$u + F_r = mg$$

- ❖ At this point the resultant force is zero and the ball attains a constant velocity called **terminal velocity**.



- ❖ **Terminal velocity** is defined as the constant velocity attained by a body falling in a fluid when the sum of upward forces is equal to the weight of the body. Graphically:


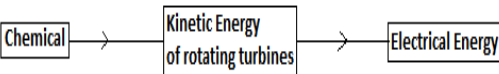
1. The diagram shows a tall measuring cylinder containing a viscous liquid. A very small steel ball is released from rest at the surface of the liquid as shown. Sketch the velocity-time graph for the motion of the ball from the time it is released to the time just before it reaches the bottom of the cylinder.

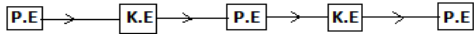


2. Two small spherical identical stones A and B are released from the same height above the ground. B falls through air while A falls through water. Sketch the graphs of velocity against time (t) for each stone. Label the graph appropriately.

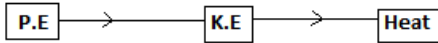
Specific objectives	Content																					
By the end of this topic, the learner should be able to: a) Describe energy transformations b) State the law of conservation of energy c) Define work, energy, power and state their SI units d) Define mechanical advantage velocity ratio and efficiency of machines e) Solve numerical problems involving work, energy, power and machines.	1. Forms of energy and energy transformations 2. Sources of energy <ul style="list-style-type: none">RenewableNon- renewable 3. Law conservation of energy 4. Work ,energy and power (work done by resolved force not required) 5. Kinetic and potential energy 6. Simple machines 7. Problems on work, energy, power and machines																					
Energy • Energy is the capacity to do work. The SI unit of energy is the <i>joule (J)</i> after <i>the physicist James Prescott Joule</i> who was also a brewer.	 • Note: <i>potential energy</i> is the energy possessed by a body due to its relative position or state while <i>kinetic energy</i> is the one possessed by a body due to its motion.																					
Sources of Energy • They are classified into renewable and nonrenewable sources. <i>i. Renewable sources</i> • These are sources whose supply can be renewed again and again for use. Examples are; <i>water, solar, wind, geothermal etc.</i> <i>ii. Non-renewable sources</i> • These are sources of energy whose supply cannot be renewed again and again for use. Examples are; <i>fossils, firewood, nuclear source etc.</i>	Conservation and Transformation of Energy <i>The Law of Conservation of Energy</i> • This law states that <i>“Energy can neither be created nor destroyed but can only be transformed from one form to another.”</i> <i>Energy Transformation</i> • Any device that facilitates the transformation of energy from one form to another is called a <i>transducer</i> . The following are some examples:																					
Forms of Energy • The various forms of energy include: <i>o Mechanical (potential and kinetic)</i> <i>o Chemical – stored in batteries and foods</i> <i>o Electrical</i> <i>o Light</i>	<table><tr><th><i>Initial form of energy</i></th><th><i>Final form of energy</i></th><th><i>Transducer</i></th></tr><tr><td>Solar</td><td>Heat</td><td>Solar panel</td></tr><tr><td>Electrical</td><td>Kinetic</td><td>Motor</td></tr><tr><td>Kinetic</td><td>Electrical</td><td>Dynamo</td></tr><tr><td>Solar</td><td>Electrical</td><td>Solar cell</td></tr><tr><td>Heat</td><td>Electrical</td><td>Thermocouple</td></tr><tr><td>electrical</td><td>Sound</td><td>Loudspeaker</td></tr></table>	<i>Initial form of energy</i>	<i>Final form of energy</i>	<i>Transducer</i>	Solar	Heat	Solar panel	Electrical	Kinetic	Motor	Kinetic	Electrical	Dynamo	Solar	Electrical	Solar cell	Heat	Electrical	Thermocouple	electrical	Sound	Loudspeaker
<i>Initial form of energy</i>	<i>Final form of energy</i>	<i>Transducer</i>																				
Solar	Heat	Solar panel																				
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electrical	Sound	Loudspeaker																				

<ul style="list-style-type: none"><i>o Nuclear</i><i>o Wave</i>	<table><tr><td>chemical</td><td>Electrical</td><td>Battery</td></tr></table> <ul style="list-style-type: none">• <i>Note:</i> Energy transformations are represented by <i>charts.</i>	chemical	Electrical	Battery
chemical	Electrical	Battery		

<u>Examples</u>	<u>Examples</u>
<p><i>Describe the energy transformation that takes place in each of the following:</i></p> <p>a) <i>A car battery is used to light a bulb</i></p>  <p>b) <i>Coal is used to generate electricity</i></p>  <p>c) <i>A pendulum bob swing to and fro</i></p>	<p>1. <i>Calculate the amount of work done by:</i></p> <p>a) <i>A machine lifting a load of mass 50 kg through a vertical distance of 2.4m</i></p> <p><u>Solution</u></p> <p>work done, $W = \text{force, } F \times \text{distance}$</p> <p>work done = $mg \times \text{distance}$</p> <p style="text-align: center;">$= (50 \times 10)\text{N} \times 2.4 \text{ m} = 1200 \text{ J}$</p> <p>b) <i>A laborer who carries a load of mass 42kg to a</i></p>



- d) *Water at the top of a waterfall falls and its temperature rises on reaching the bottom*



Work and Energy

- Work is defined as the product of force and distance moved in the direction of application of the force.

work done, $W = \text{force, } F \times \text{distance moved in}$

the direction of
the applied force, d

$$W = F \times d$$

- Work is therefore said to be done when an applied force makes the point of application of the force move in the direction of the force. No work is done when a person pushes a wall until he sweats or carrying a bag of cement on his head for hours while standing.

height of 4.0m

Solution

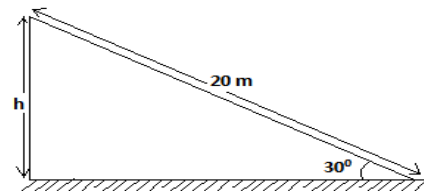
work done, $W = \text{force, } F \times \text{distance}$

work done = $mg \times \text{distance}$

$$= (42 \times 10)\text{N} \times 4.0\text{ m} = 1680\text{ J}$$

2. *A man of mass 70 kg walks up a track inclined at an angle of 30° to the horizontal. If he walks 20 m, how much work does he do?*

Solution



work done, $W = \text{force, } F \times \text{distance}$

work done = $mg \times \text{distance, } h$

$$= (70 \times 10)\text{N} \times (20 \sin 30^\circ)\text{m} = 7000\text{ J}$$

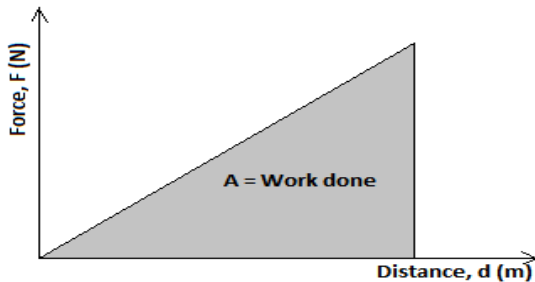
Exercise

- The **S**/unit of work is **the joule (J)**.
1 joule (J) = 1 newton metre (Nm)

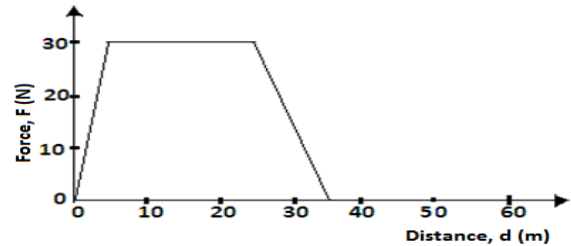
N/B: Joule is the work done when the point of application of a force of 1 newton moves through 1 metre in the direction of the force.

Notes:

- Work done is equivalent to energy converted while doing work.
- The area under force-distance graph represents work done by the force or energy converted.



- A girl of mass 40 kg walks up a flight 10 steps. If each step is 40 cm high, calculate the work done by the girl.
- A body is acted upon by a varying force F over a distance of 35 m as shown in the figure below.

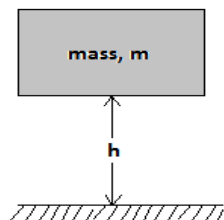


Calculate the total work done by force

- Sometimes work is not done even if there is an applied force. Describe some situations when this can happen.

Gravitational Potential Energy

- This is the energy possessed by a body due to its height above some surface. Consider a block of mass m raised through the height h from the ground. At that height the block has gravitational potential energy.



Potential energy, P.E gained = work done in raising the block P.E

$$= \text{weight of the block} \times \text{height}$$

$$\text{P.E} = mgh$$

Elastic Potential Energy

- This is the energy stored in a stretched or compressed spring. The energy is equal to work done in stretching or compressing the spring.

work done = Force \times distance moved in direction of force

work done = Average Force \times change in length of spring

$$(\text{compression or extension}) \text{ work done} = \frac{0 + F}{2} \times e;$$

$$\text{work done} = \frac{F}{2} \times e,$$

but $F = ke$, where k is the spring constant \therefore

$$\text{work done} = \left(\frac{1}{2}ke\right)e,$$

$$\text{work done} = \frac{1}{2}ke^2$$

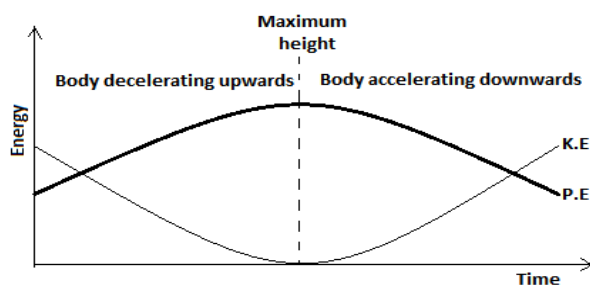
Kinetic Energy, K.E

- Consider a body of mass m being acted upon by a steady force F . the body accelerates uniformly from rest to final velocity v in time t seconds. If it covers a distance s ;

work done in accelerating the body

Variation of K.E and P.E for A Body Projected Upwards

- Consider a body of mass m projected vertically upwards. Gravitational force is the only force acting on it, assuming negligible air resistance. As it rises kinetic energy decreases since the velocity decreases (the body decelerates upwards). At the same time, the potential energy of the body increases and becomes maximum at the highest point, where K.E is zero. As the body falls from the highest point, P.E decreases while K.E increases. The curves for variation of K.E and P.E of the body with time are shown below.



- Therefore, at any given points;
total energy, $E = \text{P.E} + \text{K.E} = \text{Constant}$.

Examples

A stone of mass 2.5 kg is released from a height of 5.0 m above the ground:

- Calculate the velocity of the stone just before it strikes the ground.
- At what velocity will the stone hit the ground if a constant air resistance force of 1.0 N acts on it as it falls?

Solution

$$\text{a) } mgh = \frac{1}{2}mv^2$$

$$2.5 \times 10 \times 5.0 = \frac{1}{2} \times 2.5 \times v^2$$

$$v = \sqrt{\frac{100}{1}} = 10 \text{ ms}^{-1}$$

= the K.E gained by the body

= Force, $F \times \text{distance}, s$

Method 1

$$K.E = F \times s ;$$

$$K.E = ma \times (\text{average velocity} \times \text{time})$$

$$K.E = m \left(\frac{v-u}{t} \right) \times \left(\frac{u+v}{2} \times t \right)$$

$$K.E = m \left(\frac{v-0}{t} \right) \times \left(\frac{0+v}{2} \times t \right) = \frac{mv}{t} \times \frac{vt}{2}$$

$$K.E = \frac{1}{2}mv^2$$

Method 2

$$K.E = F \times s ; K.E = ma \times \left(ut + \frac{1}{2}at^2 \right)$$

$$K.E = m \left(\frac{v-u}{t} \right) \times \left(0 \times t + \frac{1}{2} \left(\frac{v-u}{t} \right) t^2 \right)$$

$$K.E = m \left(\frac{v-0}{t} \right) \times \left(0 \times t + \frac{1}{2} \left(\frac{v-0}{t} \right) t^2 \right) = \frac{mv}{t} \times \frac{vt}{2}$$

$$K.E = \frac{1}{2}mv^2$$

$$b) \text{ Resultant force} \times h = \frac{1}{2}mv^2 ;$$

$$(mg - \text{resistance})h = \frac{1}{2}mv^2$$

$$(2.5 \times 10 - 1.0) \times 5 = \frac{1}{2} \times 2.5 \times v^2$$

$$v = \sqrt{\frac{240}{2.5}} = 9.798 \text{ ms}^{-1}$$

Exercise

1. A stone of mass 5 kg moves through a horizontal distance 10 m from rest. If the force acting on the stone is 8 N, calculate:

- the work done by the force
- the kinetic energy gained by the stone
- the velocity of the stone

2. Calculate the amount of energy needed by a catapult to throw a stone of mass 500g with a velocity of 10 ms^{-1}

3. A tennis ball is dropped from a height of 1.8m. it rebounds to a height of 1.25m.

- Describe the energy changes which take place
- With what velocity does the ball hit the ground?
- With what velocity does the ball leave the ground?

4. A ball rolls on a table in a straight line. A part from the transitional kinetic energy, state the other form of kinetic energy possessed by the ball.

5. A body has 16 Joules of kinetic energy. What would be its kinetic energy if its velocity was double?

6. A force of 8N stretches a spring by 10cm. How much work is done in stretching this spring by 13cm?

Power

- Power is defined **as the rate of doing work (i.e. work done per unit time)**. Since work done is equivalent to energy used, and energy cannot be destroyed or created but converted from one form to another,
- Power can also be defined as **the rate of energy conversion** OR the rate of transfer of energy.

$$\text{Power} = \frac{\text{work done}}{\text{time taken}} \text{ or } \text{Power} = \frac{\text{energy converted}}{\text{time taken}}$$

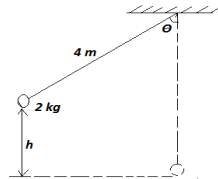
- The **SI** unit of power is **the watt**, named after **the physicist James Watt**.

$$1 \text{ watt (W)} = 1 \text{ joule per second (Js}^{-1}\text{)}$$

Relationship between power and velocity

7. A simple pendulum is released from rest and it swings towards its lowest position. If the speed at the lowest position is 1.0 m/s , calculate the vertical height of the bob when it is released.

8. A metal ball suspended vertically with a wire is displaced through an angle θ as shown in the diagram below. The ball is released from A and swing back to B.



Given that the maximum velocity at the lowest point B is 2.5 ms^{-1} . Find the height h from which the ball is released

9. A 30g bullet strikes a tree trunk of diameter 40cm at 200 ms^{-1} and leaves it from the opposite side at 100 ms^{-1} . Find:

I. The kinetic energy of the bullet just before it strikes the tree.

II. The kinetic energy of the bullet just before it leaves from the tree.

III. The average force acting on the bullet as it passes through the tree.

10. The initial velocity of a body of mass 20kg is 4 ms^{-1} . How long would a constant force of 5.0N act on the body in order to double its kinetic energy?

11. A compressed spring with a load attached to one end and fixed at the other and is released as shown below.



Sketch on the same axis the variation of potential energy, kinetic energy and total energy with time

12. The figure below shows how the potential energy (P.E) of

$$\text{Power} = \frac{\text{work done}}{\text{time taken}}$$

$$\text{Power} = \frac{\text{Force} \times \text{displacement of point of application of force}}{\text{time taken}}$$

$$\text{Power} = \text{Force} \times \frac{\text{displacement}}{\text{time taken}}$$

$$\text{Power} = \text{Force} \times \text{velocity}$$

Examples

1. An electric motor raises a 50 kg mass at a constant velocity. Calculate the power of the motor if it takes 30 seconds to raise the mass through a height of 15 m

Solution

$$\text{Power} = \text{Force} \times \text{velocity}; \text{Power} = mg \times \frac{\text{displacement}}{\text{time taken}}$$

$$\text{power} = 50 \text{ N} \times \frac{15 \text{ m}}{30 \text{ s}} = 25 \text{ W}$$

2. A soldier climbs to the top of the watch tower in 15 minutes. If the work done by the soldier against gravity is 60 kJ, what is his average power in climbing?

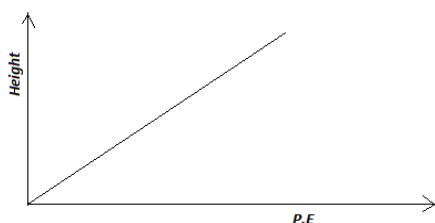
Solution

$$\text{Power} = \frac{\text{work done}}{\text{time taken}}$$

$$\text{power} = \frac{60 \times 1000}{15 \times 60} = 66.67 \text{ W}$$

Exercise

a ball thrown vertically upwards varies with height



On the same axes plot a graph of the kinetic energy of the ball

1. A crane lifts a load of 200 kg through a vertical distance of 3.0m in 6 seconds. Determine the;

- I. Work done
- II. Power developed by the crane

2. A load of 100N is raised 20m in 50s. Calculate;

- I. The gain in potential energy
- II. The power developed

3. Water falls through a height of 60m at a rate of flow of 10×10^5 litres per minute. Assuming that there are no energy losses, calculate the amount of power generated at the base of the water fall. (the mass of 1 liter of water is 1 kg)

4. If 50 litres of water is pumped through a height of 15m in 30 seconds, what is its power rating of the pump is 80% efficient? (the mass of 1 liter of water is 1 kg)

5. A small wind pump develops an average power of 50N. It raises water from a borehole to a point 12N above the water level. Determine the mass of water delivered in one hour.

Simple Machines

- A machine is a device that makes work easier OR is any device by means of which a force applied at one point of it can be used to overcome a force at some other point of it.

Terms Associated with Machines

1. Effort, E

- This is the force applied to the machine. The SI unit of effort is **the newton (N)**.

2. Load, L

- This is the force exerted by the machine. The SI unit of load is **the newton (N)**.

3. Mechanical advantage, $M.A$

- This is the ratio of the load to the effort. It has no units since it is a ratio of two forces.

- N/B Efficiency, just like M.A of a machine depends on **friction between moving parts of a machine** and **weight of the parts that have to be lifted**. These reasons explain why the efficiency of a machine is always less than 100%.

Examples

1. A certain machine uses an effort of 400N to raise a load of 600N. If the efficiency of the machine is 75%, determine its velocity ration.

Solution

$$M.A = \frac{\text{load}}{\text{effort}}$$

$$M.A = \frac{600}{400} = 1.5$$

$$\eta = \frac{M.A}{V.R} \times 100\% \Rightarrow 75 = \frac{1.5}{V.R} \times 100\%$$

$$V.R = \frac{1.5}{75} \times 100\% = 2$$

$$\text{M.A} = \frac{\text{load}}{\text{effort}}$$

- For most machines, M.A is greater than one since load is greater than effort. In a few machines M.A. is less than one (i.e effort is greater than load) e.g. a bicycle.

Factors Affecting M.A of a Machine

- Friction between moving parts of the machine*—The greater the friction, the less the mechanical advantage
- Parts of the machine that have to be lifted* – The heavier the weight, the less the mechanical advantage.

4. Velocity Ratio, V.R

- It is the ratio of the distance moved by the effort D_E to the distance moved by the load D_L in the same time.

$$\text{Velocity ratio, V.R} = \frac{\text{distance moved by the effort, } D_E}{\text{distance moved by the load, } D_L}$$

$$\text{V.R} = \frac{D_E}{D_L}$$

- Note:** If two machines A and B with velocity ratios $V.R_A$ and $V.R_B$ respectively are combined, the resultant velocity ratio V.R will be given by:

$$\text{V.R} = \text{V.R}_A \times \text{V.R}_B$$

5. Efficiency, η

- It is the ratio of the useful work done by the machine (work output) to the total work put into the machine (work input) expressed as a percentage.

$$\eta = \frac{\text{work done by the machine (work output)}}{\text{total work into the machine (work input)}} \times 100\%$$

Relationship between Mechanical Advantage, Velocity Ratio and Efficiency

$$\eta = \frac{\text{work done on load (work output)}}{\text{work done by effort (work input)}} \times 100\%$$

$$\eta = \frac{L \times D_L}{E \times D_E} \times 100\%$$

- A crane lifts a load of 200 kg through a vertical distance of 3.0m in 6 seconds. Determine the;

I. Work done

II. Power developed by the crane

III. Efficiency of the crane given that it is operated by an electric motor rated 1.25 kW.

Solution

I. work done = mgh

$$\text{work done} = 200 \times 10 \times 3.0 = 6000 \text{ J}$$

II. Power = $\frac{\text{work done}}{\text{time taken}}$

$$\text{Power} = \frac{6000}{6} = 1000 \text{ W or 1kW}$$

III. $\eta = \frac{\text{power output}}{\text{power input}} \times 100\%$

$$\eta = \frac{1000}{1250} \times 100\% = 80\%$$

Exercise

- When an electric pump whose efficiency is 70% raises water to a height of 15m, water is delivered at the rate of 350 litres per minute.

I. What is the power rating of the pump?

II. What is the energy lost by the pump per second?

- An electric pump can raise water from a lower-level reservoir to the high level reservoir to the high level reservoir at the rate of 3.0×10^5 kg per hour. The vertical height of the water is raised 360m. If the rate of energy loss in form of heat is 200 kW, determine the efficiency of the pump.

- Define the efficiency of a machine and give a reason why it can never be 100%

- A pump uses 1g of a mixture of petrol and alcohol in the ratio 4:1 by mass to raise 1000 kg of water from a well 200m deep.

I. How much energy is given by 1g of mixture?

$$\eta = \frac{L}{E} \times \frac{D_L}{D_E} \times 100\%$$

$$\text{but } \frac{L}{E} = \text{M.A. and } \frac{D_L}{D_E} = \frac{1}{\text{V.R.}}$$

$$\eta = \text{M.A.} \times \frac{1}{\text{V.R.}} \times 100\%$$

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} \times 100\%$$

II. If the pump is 40% efficient, what mass of this mixture is needed to raise the water?
(1g of alcohol = 7000J, of petrol = 48000J)

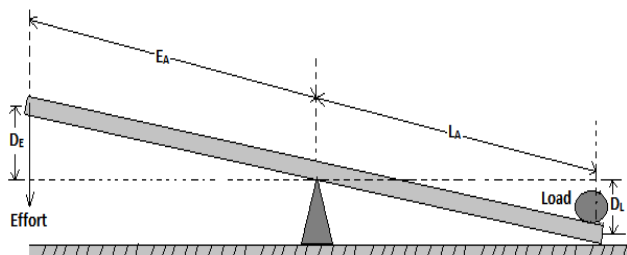
5. In a machine, this load moves 2m when the effort moves 8m, if an effort of 20N is used to raise a load of 60N, what is the efficiency of the machine?

Types of Simple Machines

1. Levers

- A lever is a simple machine whose operation relies on the **principle of moments**. It consists of the **effort arm**, **load arm** and **pivot** **fulcrum**. **The effort arm**, E_A is the perpendicular distance of the line of action of the effort from the pivot. **The load arm**, L_A is the perpendicular distance of the line of action of the load from pivot.

- Consider the figure of simple levers shown below



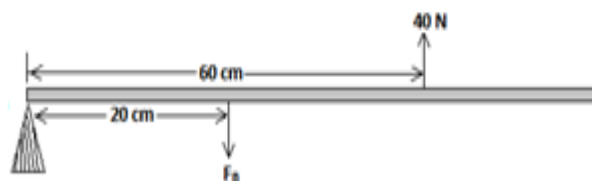
$$\text{V.R. of lever} = \frac{\text{distance moved by the effort, } D_E}{\text{distance moved by load, } D_L}$$

$$\text{Using concept of similar triangles; } \frac{D_E}{D_L} = \frac{E_A}{L_A}$$

Exercise

The figure below shows a lever

- Determine the force F_A in each case
- Determine the M.A and V.R in each case
- Calculate efficiency in each case



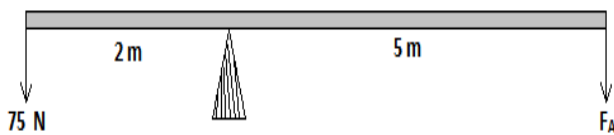
Classes of Levers

- Levers with pivot between load and effort e.g. pliers, hammer etc.
- Levers with the load between pivot and effort e.g. wheel barrow, bottle openers etc.
- Levers with the effort between the load and the pivot e.g. sweeping brooms, a fishing rod and hammer arm.

Therefore, for the lever, $V.R = \frac{\text{effort arm, } E_A}{\text{load arm, } L_A}$

Example

The figure below shows a lever



- Determine the force F_A
- Determine the M.A and V.R
- Calculate efficiency

Solution

a) clockwise moment = anticlockwise moment

$$F_A \times 5 = 75 \times 2$$

$$\Rightarrow F_A = \frac{150}{5} = 30 \text{ N}$$

b) $M.A = \frac{\text{load}}{\text{effort}};$

$$M.A = \frac{75}{30} = 2.5$$

$$V.R = \frac{\text{effort arm, } E_A}{\text{load arm, } L_A};$$

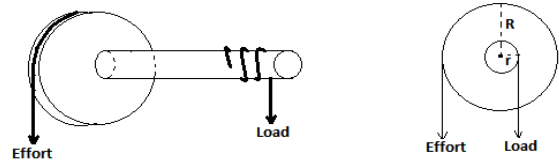
$$V.R = \frac{5}{2} = 2.5$$

c) $\eta = \frac{M.A}{V.R} \times 100\%;$

$$\eta = \frac{2.5}{2.5} \times 100\% = 100\%$$

2. Wheel and axle

- Consists of **large wheel of radius R** attached to **axle of radius r** , the effort is applied on the wheel while the load is attached to the axle. An example of the wheel and axle is the **winch** used to draw water from well



- In one complete revolution the wheel moves through a distance $2\pi R$ while the load moves through $2\pi r$.

$$V.R \text{ of wheel and axle} = \frac{2\pi R}{2\pi r} = \frac{R}{r}, \quad V.R = \frac{\text{Radius of wheel}}{\text{Radius of axle}}$$

Example

A wheel and axle is used to raise a load of 300N by a force of 50N applied to the rim of the wheel. If the radii of the wheel and axle are 85cm and 10cm respectively, calculate the M.A, V.R and efficiency.

Solution

$$M.A = \frac{\text{load}}{\text{effort}};$$

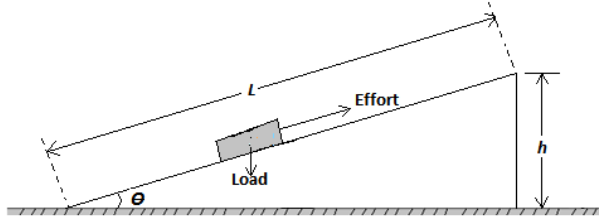
$$M.A = \frac{300}{50} = 6$$

$$V.R = \frac{\text{Radius of wheel}}{\text{Radius of axle}}; \quad V.R = \frac{0.85 \text{ m}}{0.10 \text{ m}} = 8.5$$

$$\eta = \frac{M.A}{V.R} \times 100\%; \quad \eta = \frac{6}{8.5} \times 100\% = 70.59\%$$

3. The Inclined Plane

- Consider the plane below inclined at an angle θ to the horizontal.



$$\text{V.R of the inclined plane} = \frac{\text{effort distance}}{\text{load distance}} = \frac{L}{h}$$

$$\text{but } \sin \theta = \frac{h}{L} \Rightarrow h = L \sin \theta$$

$$\text{V.R of the inclined plane} = \frac{L}{L \sin \theta}; \quad \text{V.R} = \frac{1}{\sin \theta}$$

Example

A man uses the inclined plane to lift 100kg load through a vertical height of 8.0m. The inclined plane makes an angle of 40° with the vertical. If the efficiency of the inclined plane is 85%, calculate

- The effort needed to move the load up the inclined plane at a constant velocity.
- The work done against friction in raising the load through the height of 8.0m

Solution

$$\text{I. } \eta = \frac{\text{M.A}}{\text{V.R}} \times 100\%; \quad 85 = \frac{\text{M.A}}{1/\sin 50} \times 100\%;$$

$$\text{M.A} = \frac{85 \times 1/\sin 50}{100} = 1.110$$

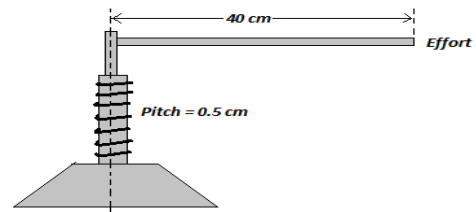
$$\text{M.A} = \frac{\text{load (mg)}}{\text{effort}}; \quad 1.110 = \frac{1000}{\text{effort}}$$

$$\text{V.R of screw} = \frac{\text{circumference of the screw head}}{\text{pitch}} = \frac{2\pi R}{\text{pitch}}$$

Where R is the radius of the screw head. A screw combined with a lever is used as a jack for lifting heavy loads such as cars.

Example

The figure below shows a car-jack with a lever arm of 40 cm and a pitch of 0.5 cm. If the efficiency is 60 %, what effort would be required to lift a load of 300 kg.



Solution

$$\eta = \frac{\text{M.A}}{\text{V.R}} \times 100\% \Rightarrow \eta = \frac{\text{M.A}}{2\pi R/\text{pitch}} \times 100\%$$

$$60 = \frac{\text{M.A}}{2\pi \times 0.4/0.005} \times 100\%$$

$$\Rightarrow \text{M.A} = \frac{60 \times (2\pi \times 0.4)/0.005}{100} = 301.63$$

$$\text{M.A} = \frac{\text{Load(mg)}}{\text{Effort}} \Rightarrow 301.63 = \frac{300 \times 10}{\text{Effort}}$$

$$\text{Effort} = \frac{300 \times 10}{301.63} = 9.46 \text{ N}$$

Exercise

A car weighing 200kg is lifted with a jack of 15mm pitch. If the handle is 32cm from the screw, find the applied force.

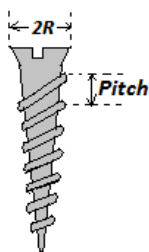
$$\text{effort} = \frac{1000}{1.110} = 900.9 \text{ N}$$

II. 15 % of work input is used to overcome friction

i.e. 15 % of (effort \times distance moved by effort)

$$\frac{15}{100} \times \left(900.9 \times \frac{8}{\sin 50} \right) = 1411.26 \text{ joules}$$

4. The Screw



- The distance between two successive threads of a screw is called the **pitch of the screw**.
- In one complete revolution, the screw moves forward or backward through a distance equal to one pitch.

Exercise

A certain gear has 30 teeth and drives another with 75 teeth. How many revolutions will the driver gear make when the driven gear makes 100 revolutions?

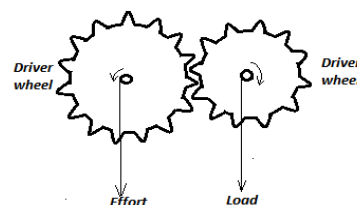
6. Pulleys

- A pulley is a wheel with a groove for accommodating a string or rope. **V.R for a pulley system is the number of ropes supporting load.** There are three common pulley systems.

A) Single Fixed Pulley

5. Gears

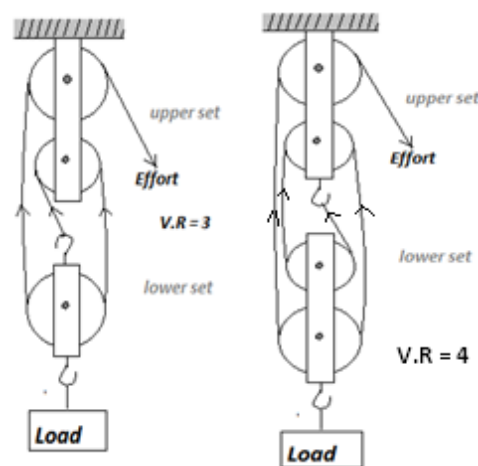
- A gear is a wheel which can rotate about its centre and has equally spaced teeth or cogs around it. Two or more gears are arranged to make a machine which can be used to transmit motion from one wheel to another.



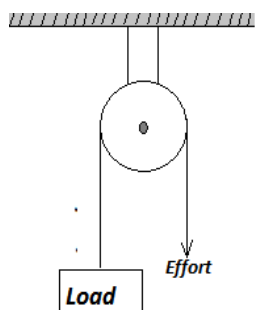
- If the driver wheel has **P** teeth and the driven wheel **Q** teeth, then, when the driver wheel makes one revolution, the driven wheel makes $\frac{P}{Q}$ revolution

$$\text{V.R} = \frac{\text{revolutions made by the driver wheel}}{\text{revolution made by the driven wheel}}; \quad \frac{1}{\frac{P}{Q}} = \frac{Q}{P}$$

$$\text{V.R of a gear system} = \frac{\text{number of teeth in driven wheel}}{\text{number of teeth in the driver wheel}}$$

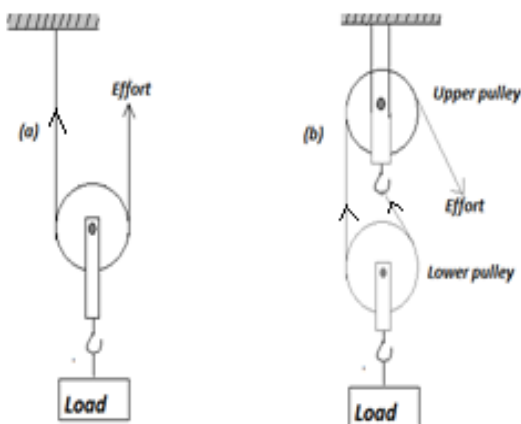


NB For a block and tackle with an odd number of pulleys it is convenient to have more pulleys fixed



The **velocity ratio** of single fixed pulley is **1**.

B) Single Movable Pulley



The **upper pulley** in (b) makes it possible for effort to be applied downward. The **velocity ratio** of each of the pulleys (a) and (b) is **2** since two ropes are supporting the load.

C) Block and tackle

This system consists of two or more sets of pulley blocks. Below are examples.

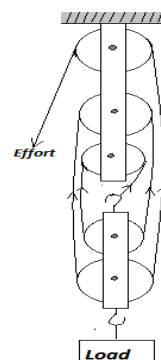
than movable.

Example

A block and tackle system is used to lift a mass of 400 kg. If this machine has a velocity ratio of 5 and an efficiency of 75 %

a) Sketch a possible arrangement of the pulleys showing how the rope is wound

• Solution



b) Calculate the effort applied.

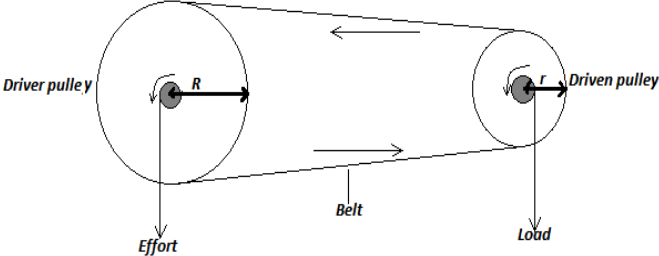
• Solution

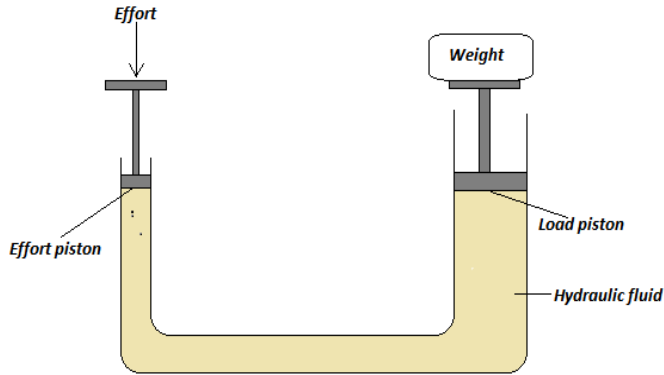
$$\eta = \frac{M.A}{V.R} \times 100\% \Rightarrow 75 = \frac{M.A}{5} \times 100\%$$

$$M.A = \frac{75 \times 5}{100} = 3.75$$

$$M.A = \frac{\text{Load}}{\text{Effort}}; 3.75 = \frac{400 \times 10}{\text{Effort}}$$

$$\Rightarrow \text{Effort} = \frac{4000}{3.75} = 1066.7 \text{ N}$$

	<p style="text-align: center;"><u>Exercise</u></p> <ol style="list-style-type: none"> 1. An effort of 125N is used to lift a load of 500N through a height of 2.5m using a pulley system. If the distance moved by the effort is 15m, calculate <ol style="list-style-type: none"> a) The work done on the load b) The work done by the effort c) The efficiency of the pulley system. 2. Draw a lock and tackle pulley system of V.R 6 to show how the pulley can be used to raise a load L by applying an effort E
<p>7. Pulley Belts</p>  <ul style="list-style-type: none"> • They are found in posh mills, sewing machines, motor engines etc. If the radius of the driver pulley is R and that of the driven pulley is r, the belt turns a distance of $2\pi R$ when the driving wheel makes the revolution. The load wheel (driven wheel) at the same time makes $\frac{2\pi R}{2\pi r} = \frac{R}{r} \text{ revolutions}$ $V.R = \frac{\text{number of revolutions made by effort}}{\text{number of revolutions made by load}} = \frac{1}{\frac{R}{r}} = \frac{r}{R}$ $V.R = \frac{\text{radius of driven wheel}}{\text{radius of driver wheel}}$ <p>8. Hydraulic machines</p>	<p style="text-align: center;"><u>Example</u></p> <p>The radius of the effort piston of a hydraulic lift is 2.1cm while that of the load piston is 8.4cm. This machine is used to raise a load of 180kg at a constant velocity through a height of 5m. Given that the machine is 75% efficient, calculate:</p> <ol style="list-style-type: none"> a) The effort needed b) The energy wasted in using this machine. <p><u>Solution</u></p> <p>(a) $\eta = \frac{M.A}{V.R} \times 100\%$ but,</p> $V.R = \frac{\text{cross-section area of load piston}}{\text{cross-section area of effort piston}} = \frac{\pi R_L^2}{\pi R_E^2}$ $\frac{r \times 0.084^2}{r \times 0.021^2} = 16$ $75 = \frac{M.A}{16} \times 100\% \Rightarrow M.A = \frac{75 \times 16}{100} = 12$ $E = \frac{L}{M.A} \Rightarrow E = \frac{180 \times 10}{12} = 150 \text{ N}$ <p>(b) 25 % of work input is equivalent to energy wasted, i.e. 15 % of ((work output)/Efficiency $\times 100$)</p> $\frac{25}{100} \times \left(\frac{1800 \times 5}{75} \times 100 \right) = 3000 \text{ N}$ <p style="text-align: center;"><u>Exercise</u></p>



When the effort piston moves downwards the load piston is pushed upwards.

volume of liquid that leaves effort cylinder
= volume of liquid that enters load cylinder

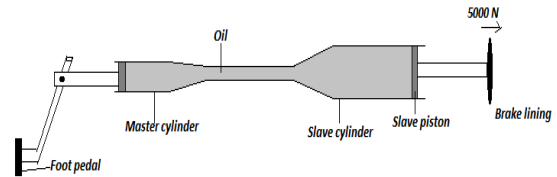
distance moved by effort \times cross - section of effort piston
= distance moved by load \times cross - section area of piston

distance moved by effort / distance moved by load = cross - section area of load piston / cross - section area of effort piston

$$\text{Velocity ratio} = \frac{2\pi R^2}{2\pi r^2} = \frac{R^2}{r^2}$$

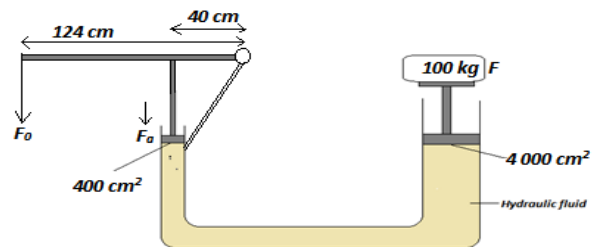
Revision Exercise

1. The diagram below shows hydraulic brake system.



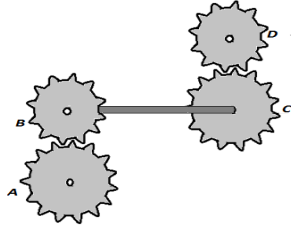
- State three properties of the hydraulic brake oil.
- A force of 20N is applied on a foot pedal to a piston of areas 50cm^2 and this causes stopping force of 5000N.
 - Pressure in the master cylinder
 - Area of the slave piston
 - Velocity ratio of the system

2. Study the figure below of a hydraulic lift and answer the question below.

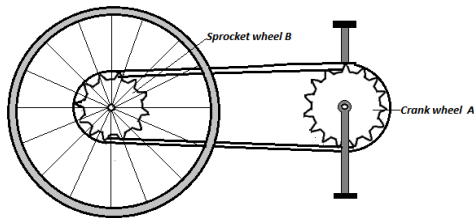


The areas of cross - sections of the pistons and the length of the arm are as indicated. Find F_o , M.A and efficiency of the machine.

1. The figure below shows a system of gears for transmitting power. Gear A has 200 teeth and act as the driving gear. Gears B and C with 40 teeth and 100 teeth respectively are mounted on the same axle and they transmit motion to the last gear D which has 50 teeth.



- I. In what direction(s) would gear C and D rotate? If gear A is rotated in clockwise direction.
 - II. Find the velocity ratio of the gear system
2. The figure below shows the rear wheel of a bicycle and the crank wheel A, connected to the sprocket B by a chain. If wheel A has 40 teeth while B has 25 teeth and the radius of the rear wheel is 42 cm, calculate:
- a) the velocity ratio of the machine,
 - b) the distance travelled by the bicycle in one revolution of the crank wheel.

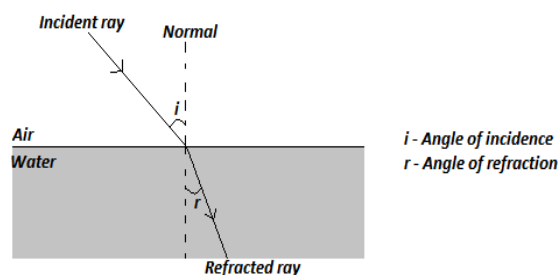


3. The pulley system above has a MA OF 3 calculate: i) the total work done when a load of 60N is raised through a height of 9m. ii) the efficiency of the machine.

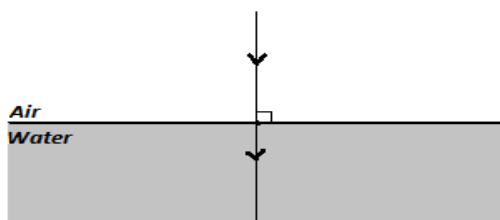
<u>Specific Objectives</u>	<u>Content</u>
<p>By the end of this topic the learner should be able to:</p> <ol style="list-style-type: none"> Describe simple experiments to illustrate refraction of light State the laws of refraction of light Verify Snell's law Define refractive index Determine experimentally the refractive index Describe experiment to illustrate dispersion of white light Explain total internal reflection and its effects State the applications of total internal reflection Solve numerical problems involving refractive index and critical angle. 	<ol style="list-style-type: none"> Refraction of light – laws of refraction (experimental treatment required) Determination of refractive index <ul style="list-style-type: none"> Snell's law Real/ apparent depth Critical angle Dispersion of white light (Experimental treatment is required) Total internal reflection and its effects: critical angle Applications of total internal reflection <ul style="list-style-type: none"> Prism periscope Optical fibre Problems on refractive index and critical angle

Definition of Refraction of Light

- Refraction of light refers to the change in direction of light *at the interface* as it travels from one medium to another *at an angle*, for example, a ray of light from air to water. The cause of refraction of light is the change in velocity of light as it travels from one medium to another. The change in velocity is due to variation of **optical density** of media.

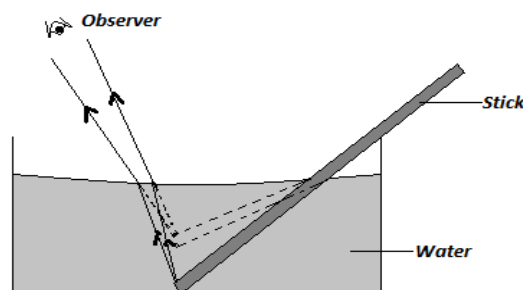


- A ray that travels perpendicular to interface proceeds across the interface not deviated since the angle of incidence to the normal is zero.



- Refraction of light is the reason as to why;

- a) a stick appears bent when part of it is in water

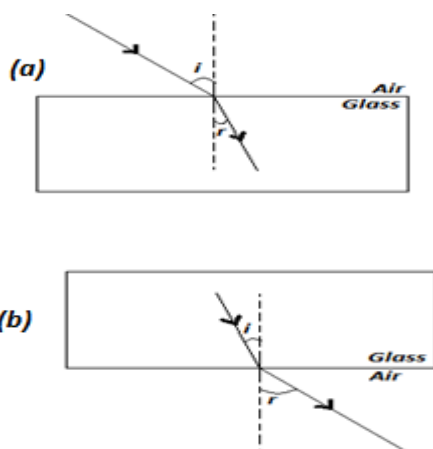


- b) a coin in a beaker of water appears near the surface than it actually is,
- c) a pool of water appears more shallow when viewed more obliquely etc. (*Students to perform this practically*)

Optical density (transmission density) and refraction of light

- A ray of light travelling from an optically less dense medium to an optically denser medium bends towards the normal after refraction e.g. a ray from air to glass block as in (a) below. The angle of incidence in this case is greater than angle of refraction.

- A ray of light travelling from an optically dense medium to an optically less dense medium bends away from the normal after refraction e.g. a ray from glass to air as in (b) below. The angle of incidence in this case is less than angle of refraction.



- NB.** some media are physically denser but optically less dense than others e.g. kerosene is physically less dense but optically denser than water.

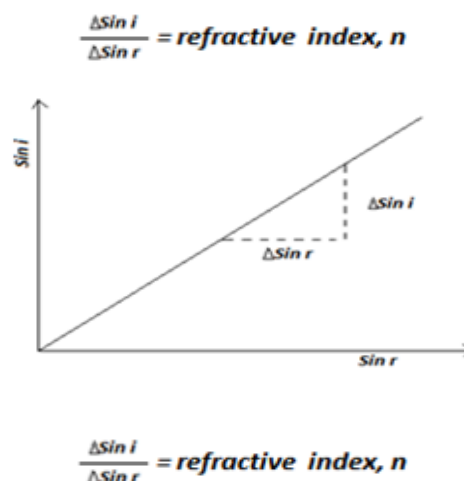
Exercise

- Define the term refraction
- Draw a diagram to show refraction for a ray of light across the following boundaries in the order they appear
 - air- water
 - water- glass
 - glass- air
 - glass- air- water
- The figure below shows how refraction occurs.

$$\frac{\sin i}{\sin r} = \text{constant.}$$

The **constant** is called **refractive index**.

- A graph of **sin i** against **sin r** is a straight line passing through the origin and slope of the graph gives the refractive index of a material.



Refractive Index, n

- Refractive index is **defined as:** the ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) for a ray passing from one medium to another.

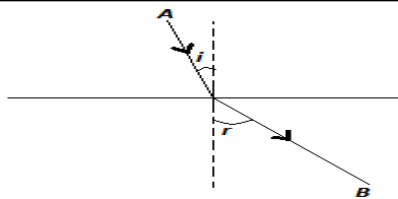
Consider a ray of light travelling from medium 1 to medium 2.

- For the two media,

$$\frac{\sin i}{\sin r} = \text{refractive index, } n_2$$

(read as the refractive index of medium 2 with respect to medium 1)

- By the principle of reversibility of light, a ray travelling from medium 2 to medium 1 along the same path is



Which of the two media is optically denser?
Explain.

4. Explain with the help of a diagram why pencil placed partly in water appears bent.

Laws of Refraction

Law 1

- The incident ray, the refracted ray and the normal all lie in the same plane at the point of incidence.

Law 2 (Snell's law)

- It states that; *"the ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) is a constant for a given pair of media"*.

Note:

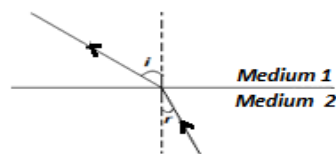
Absolute refractive index is the refractive index of a material with respect to vacuum. A vacuum has a refractive index of 1.000.

Examples

1. A ray of light passes through air into a certain transparent material. If the angles of incidence and refraction are 60° and 35° respectively, calculate the refractive index of the material

Solution

refracted making the same angles.



$$\frac{\sin r}{\sin i} = {}_2n_1. \text{ r is the angle of incidence in this case.}$$

$$\text{but, } \frac{\sin i}{\sin r} = {}_1n_2 = \frac{1}{\frac{\sin r}{\sin i}}$$

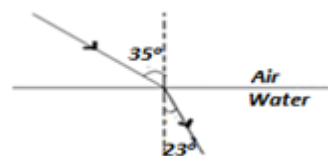
$${}_1n_2 = \frac{1}{{}_2n_1}$$

$$\frac{\sin r}{\sin i} = {}_2n_1. \text{ r is the angle of incidence in this case.}$$

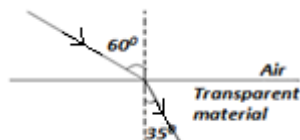
$$\text{but, } \frac{\sin i}{\sin r} = {}_1n_2 = \frac{1}{\frac{\sin r}{\sin i}}$$

$${}_1n_2 = \frac{1}{{}_2n_1}$$

3. Use the information given in the figures (a) and (b) below to calculate the refractive index ${}_a n_w$ and the angle θ



4. State the principle of reversibility of light.

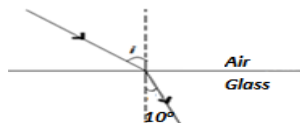


$$\text{Refractive index} = \frac{\sin i}{\sin r}$$

$$\text{Refractive index} = \frac{\sin 60}{\sin 35} = 1.510$$

2. Given that refractive index of glass is 1.5, calculate the angle of incidence for a ray of light travelling from air to glass if the angle of refraction is 10° .

Solution



$$\text{Refractive index} = \frac{\sin i}{\sin r}$$

$$1.5 = \frac{\sin i}{\sin 10^\circ}$$

$$\text{Angle of incidence, } i = \sin^{-1}(1.5 \times \sin 10^\circ) = 15.09^\circ$$

3. Calculate the refractive index for light travelling from glass to air given that $n_g = 1.572$.

Solution

$${}_g n_a = \frac{1}{{}_a n_g}$$

$${}_g n_a = \frac{1}{1.572} = 0.6361$$

Exercise

Refractive Index in Terms of Velocity

- Light travels faster in an optically less dense medium than in an optically denser medium. Consider a ray of light crossing the boundary from **medium 1** with **speed v_1** to **medium 2** with **speed v_2** , where v_1 is greater than v_2 as shown below.



- Refractive index, ${}_1 n_2$ of medium 2 with respect to medium 1 is given as:

$${}_1 n_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

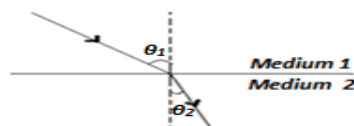
$${}_1 n_2 = \frac{\text{velocity of lighth in medium 1}}{\text{velocity of lighth in medium 2}}$$

- Absolute refractive index **n** is the refractive index of the medium when light is travelling from the vacuum to the medium.

Absolute refractive index,

$$n = \frac{\text{velocity of lighth in vacuum}}{\text{velocity of lighth in medium}}$$

- Consider the diagram below;

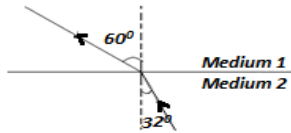


- If v_1 is the velocity of light in medium 1 of refractive index n_1 and v_2 the velocity of light in medium 2 of refractive index n_2 then;

$${}_1 n_2 = \frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2};$$

$$\Rightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2;$$

1. A ray of light striking a transparent material is refracted as shown below.



Calculate the refractive indices:

a) n_2

b) $2n_1$

2. Calculate the angle of refraction for a ray of light striking an air glass interface making an angle of 50° with the interface ($n_{\text{air}} = 1.526$)

Examples

1. Given that the refractive index of diamond is 2.51 and the velocity of light in air is $3.0 \times 10^8 \text{ ms}^{-1}$, calculate the velocity of light in diamond.

Solution

$$\text{refractive index of diamond} = \frac{\text{velocity of light in air}}{\text{velocity of light in diamond}}$$

$$2.51 = \frac{3.0 \times 10^8 \text{ ms}^{-1}}{\text{velocity of light in diamond}};$$

$$\Rightarrow \text{velocity of light in diamond} = \frac{3.0 \times 10^8 \text{ ms}^{-1}}{2.51} = 1.195 \times 10^8 \text{ ms}^{-1}$$

2. Given that the velocity of light in water is $2.27 \times 10^8 \text{ ms}^{-1}$ and in glass is $2.1 \times 10^8 \text{ ms}^{-1}$, calculate angle θ below.



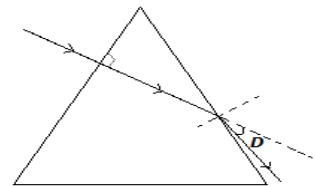
Solution

$$\frac{\sin \theta_{\text{water}}}{\sin \theta_{\text{glass}}} = \frac{v_{\text{water}}}{v_{\text{glass}}}$$

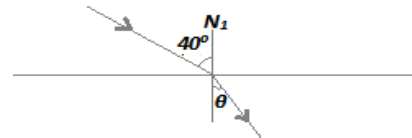
$$\Rightarrow n \sin \theta = \text{constant}$$

- Therefore, the product of the refractive index of a medium and the angle a ray makes with the normal in the medium is a constant.

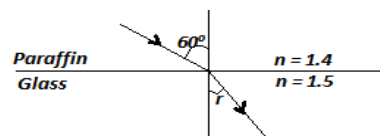
5. The figure below shows a glass prism of refractive index 1.5 with equilateral triangle cross section. Find the angle of deviation D .



6. The speed of light in medium m_1 is $2.0 \times 10^8 \text{ ms}^{-1}$ and the medium m_2 $1.5 \times 10^8 \text{ ms}^{-1}$. Calculate the refractive index of medium m_2 with respect to m_1
7. Calculate angle θ below, given that refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively. Ray is from water to glass



8. A ray of light is incident on a paraffin glass interface as shown in the figure below. Calculate r .

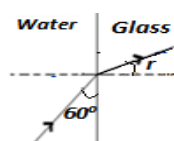


$$\frac{\sin \theta}{\sin 22} = \frac{2.27 \times 10^8}{2.1 \times 10^8};$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{2.27 \times 10^8}{2.1 \times 10^8} \times \sin 22 \right)$$

$$= 23.89^\circ$$

3. A ray of light is incident on a water-glass interface as shown below. Calculate r . (take refractive indices of glass and water $\frac{3}{2}$ and $\frac{4}{3}$ respectively)



Solution

$$n_w \sin \theta_w = n_g \sin \theta_g$$

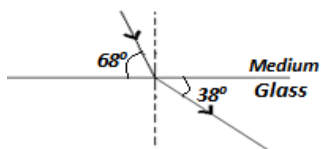
$$\frac{4}{3} \sin 40 = \frac{3}{2} \sin r$$

$$r = \sin^{-1} \left(\frac{\frac{4}{3} \sin 40 \times 2}{3} \right)$$

$$= 34.85^\circ$$

Exercise

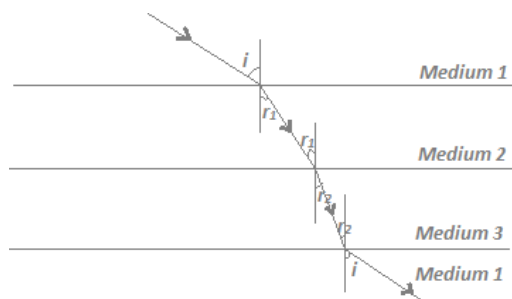
9. If the refractive index of glass is $\frac{3}{2}$, calculate the refractive index of the medium in the figure below



10. Explain why the light bends when it travels from one

Refraction through successive media

- Consider a ray of light travelling through multiple layers of transparent media whose boundaries are parallel to each other as shown below.



$${}_1n_2 = \frac{\sin i}{\sin r_1}; {}_2n_3 = \frac{\sin r_1}{\sin r_2}$$

$${}_1n_2 \times {}_2n_3 = \frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} = \frac{\sin i}{\sin r_2} \dots \dots \dots (1)$$

$${}_3n_1 = \frac{\sin r_2}{\sin i}; \Rightarrow {}_1n_3 = \frac{\sin i}{\sin r_2} \dots \dots \dots (2)$$

medium to another.

- Therefore, from equations (1) and (2); ${}_1n_3 = {}_1n_2 \times {}_2n_3$

For k successive media arranged with boundaries

parallel, ${}_1n_k = {}_1n_2 \times {}_2n_3 \dots \dots \dots k - {}_1n_k$

Examples.

- The refractive index of water is $\frac{4}{3}$ and that of glass $\frac{3}{2}$. Calculate the refractive index of glass with respect to water.

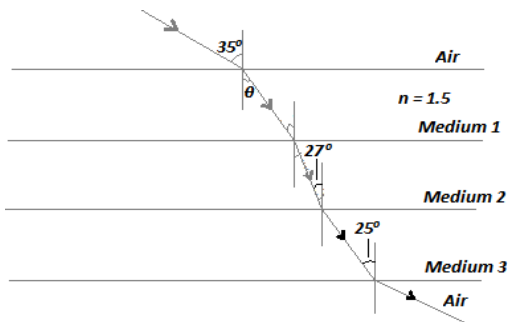
Solution

$${}_wn_g = {}_wn_a \times {}_an_g$$

$${}_wn_g = \frac{1}{\frac{4}{3}} \times \frac{3}{2};$$

$${}_wn_g = \frac{3}{4} \times \frac{3}{2} = 1.125$$

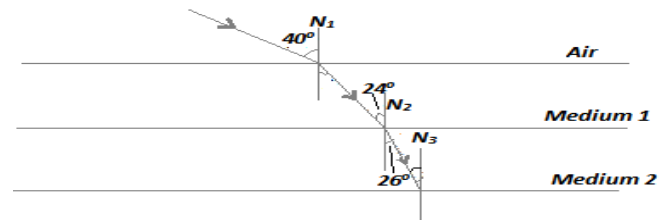
- A ray of light travels from air through multiple layers of transparent media 1, 2 and 3 whose boundaries are parallel as shown in the figure below.



Calculate :

Exercise

- A ray of light travels from air into medium 1 and 2 as shown.



Calculate;

- The refractive index of medium 1
 - Critical angle of medium 1
 - The refractive index of medium 2 relative to medium (1 n_2)
- A ray of light from air travels successively through parallel layers of water, oil, glass and then into air again. The refractive indices of water, oil, and glass are $\frac{4}{3}$, $\frac{6}{5}$ and $\frac{3}{2}$ respectively the angle of incidence in air is 60%

a) Draw a diagram to show how the ray passes through the multiple layers

b) Calculate:

- The angle of refraction in water
- The angle of incidence at the oil glass interface

a) Angle θ

b) The refractive index of m_2

c) Speed of light in m_1 (speed of light in air
 $= 3.0 \times 10^8 \text{ms}^{-1}$)

d) The refractive index of m_3 with respect to m_1

Solutions

$$a) n_a \sin \theta_a = n_1 \sin \theta_1$$

$$1 \sin 35 = 1.5 \sin \theta$$

$$\theta = \sin^{-1} \left(\frac{1 \sin 35}{1.5} \right) = 22.48^\circ$$

$$b) n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.5 \sin 22.48 = n_2 \sin 27$$

$$n_2 = \frac{1.5 \sin 22.48}{\sin 27}$$

$$= 1.263$$

$$c) \frac{\sin \theta_a}{\sin \theta_1} = \frac{v_a}{v_1}$$

$$\frac{\sin 35}{\sin 22.48} = \frac{3.0 \times 10^8}{v_1}$$

$$v_1 = \frac{\sin 22.48 \times 3.0 \times 10^8}{\sin 35}$$

$$= 2.0 \times 10^8$$

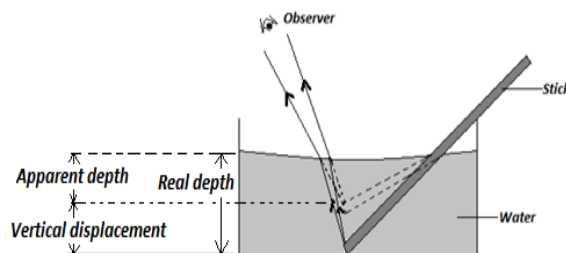
$$d) n_3 = \frac{n_2 \sin \theta_2}{\sin \theta_3};$$

$$\Rightarrow n_3 = \frac{1.263 \sin 27}{\sin 25} = 1.357$$

$$1n_3 = 1n_2 \times 2n_3$$

Refractive Index in Terms of Real and apparent depth

- An object under water or glass block appears to be nearer the surface than it actually is when viewed normally or almost normally. This is due to refraction of light.
- Real depth** is the actual depth of the object in the medium while **apparent depth** is the virtual depth of the object in the medium. The difference between real depth and apparent depth is called **vertical displacement**.



- Refractive indices of materials can be expressed in terms of real and apparent depths.

$$\text{Refractive index, } n = \frac{\text{real depth}}{\text{apparent depth}}$$

Condition for Use of the Formula. This formula applies only when the object is viewed normally.

$$n_3 = \frac{\sin 22.48}{\sin 27} \times \frac{\sin 27}{\sin 25} = 0.9040$$

Examples.

below

1. A coin in a glass jar filled with water appears to be 24.0cm from the surface of water. Calculate:

I. The height of the water in the jar, given that refractive index of water is $\frac{4}{3}$.

II. Vertical displacement

Solution

$$\text{I. Refractive index, } n = \frac{\text{real depth}}{\text{apparent depth}}$$

$$\frac{4}{3} = \frac{\text{real depth}}{24}$$

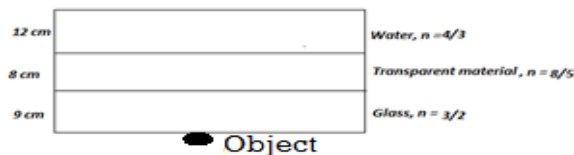
$$\text{Real depth} = \frac{4}{3} \times 24 = 32 \text{ cm.}$$

This is the height of water in jar.

II. Vertical displacement = real depth - apparent depth

$$\text{Vertical displacement} = (32 - 24)\text{cm} = 8 \text{ cm}$$

2. Calculate the displacement and apparent depth of the object shown in the figure below assuming that the object is viewed normally and boundaries of the media are parallel.



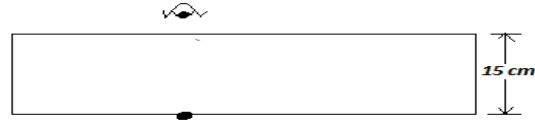
Solution

$$\text{apparent depth} = \frac{\text{real depth}}{\text{Refractive index, } n}$$

$$\text{Total apparent depth} = \frac{12}{\frac{4}{3}} + \frac{8}{\frac{8}{5}} + \frac{9}{\frac{3}{2}} = 20 \text{ cm}$$

$$\text{Vertical displacement} = \text{total real depth} - \text{total apparent depth}$$

$$\text{Vertical displacement} = (12 + 8 + 9)\text{cm} - 20 \text{ cm} = 20 \text{ cm}$$



If the speed of light in the material is $1.25 \times 10^8 \text{ ms}^{-1}$ calculate:

a) The apparent depth of the mark

b) The vertical displacement of the mark (speed of light in air = $3.0 \times 10^8 \text{ ms}^{-1}$)

- 5) A pin is placed at the bottom of a tall parallel sided glass jar containing a transparent liquid when viewed normally from the top, the pin appears nearer the surface than it actually is:

With the aid of diagram, explain this observation

- 6) The table below shows the results obtained when such an experiment was carried out using various depths of the liquid

Real depth (cm)	4.0	6.0	8.0	10.2	12.8
Apparent depth (cm)	2.4	3.6	4.8	6.10	7.32

a) Plot a graph of apparent depth against real depth

b) Using the graph, determine the refractive index of the liquid

c) What is the vertical displacement of the pin when the apparent depth is 1.22cm?

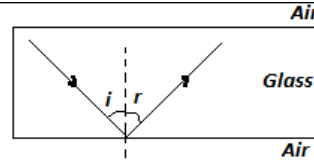
- 7) How long does it take a pulse of light to pass through a glass block 15cm in length? (Refractive index of glass is 1.5 and velocity of light in air is $3.0 \times 10^8 \text{ ms}^{-1}$)

Total Internal Reflection

- Total internal reflection refers to the complete bouncing off of light at the boundary between two media in the optically denser medium. The laws of reflection are obeyed in total internal reflection.

Exercise

- 1) A tank full of water appears to be 0.5m deep. If the height of water in the tank is 1.0m, calculate the refractive index of water.
- 2) A glass block of thickness 12cm is placed on a mark drawn on a plain paper. The mark is viewed normally through the glass. Calculate the apparent depth of the mark hence the vertical displacement (refractive index of glass = $\frac{3}{2}$)
- 3) A beaker placed over a coin contains a block of glass of thickness 12cm. over this block is water of depth 20cm. calculate the vertical displacement of the coin and hence, its apparent depth if it is viewed normally. Assume the boundaries of the media are parallel and take refractive indices of water and glass to be $\frac{4}{3}$ and $\frac{3}{2}$ respectively
- 4) A mark on a paper is viewed normally through a rectangular block of a transparent material as shown



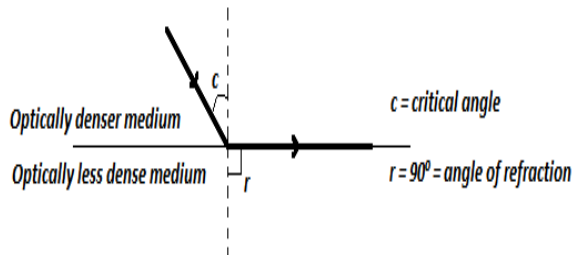
i = angle of incidence
 r = angle of reflection
 $i = r$

Conditions for Total Internal Reflection

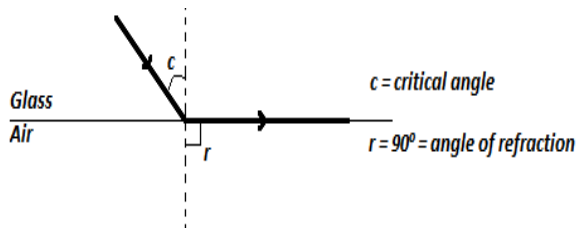
- For internal reflection to occur:
 - a) Light must be travelling from optically denser to optically less dense medium.
 - b) The angle of incidence in the optically denser medium must be greater than the critical angle

Critical Angle

Critical angle is the angle of incidence in optically denser medium for which the angle of refraction in the optically less dense medium is 90°

**Relationship between Critical Angle and Refractive Index**

- Consider a ray of light striking a glass-air interface as shown below



From Snell's law,

Solution

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$2.4 \sin c = 1.467 \sin 90$$

$$c = \sin^{-1} \left(\frac{1.467 \sin 90}{2.4} \right) = 37.66^\circ$$

Exercise

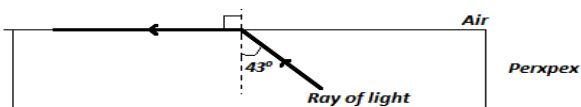
1. What do you understand by the term total internal reflection?
2. State the conditions necessary for total internal reflection
3. Define critical angle. Derive an expression for the relationship between critical angle and refractive index
4. The figure below shows a plane mirror at 30° to face of a right angled isosceles prism of refractive index 1.50. Complete the path of light ray after reflection.

- $g\eta_a = \frac{\sin c}{\sin 90^\circ}$
- But $a\eta_g = 1/g\eta_a = \frac{\sin 90^\circ}{\sin C}$
- $\sin 90^\circ = 1$
- $a\eta_g = \frac{1}{\sin C}$

Thus, refractive index, $n = \frac{1}{\sin C}$

Examples

1. The figure below shows the path of a ray of light passing through a rectangular block of Perspex placed in air



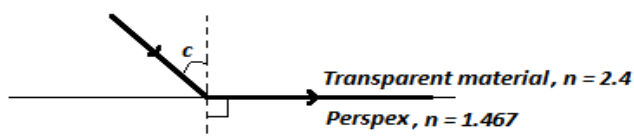
Calculate the refractive index of Perspex

Solution

$$n = \frac{1}{\sin c}$$

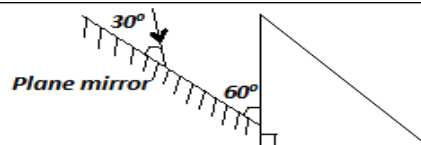
$$n = \frac{1}{\sin 43} = 1.467$$

2. A ray of light travels from a transparent medium into Perspex as shown in the figure



i. Which of the two media is optically denser?

Transparent material

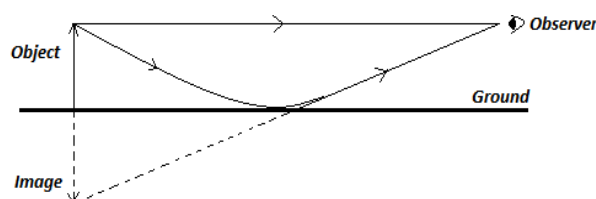


5. Calculate critical angle for diamond-water interface ($a\eta_w = 1.33$, $a\eta_d = 2.46$)

Effects of Total Internal Refraction

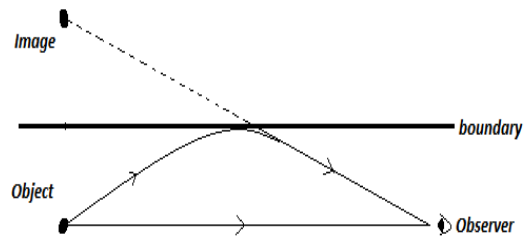
1. Mirage

- Mirage refers to optical illusion of an inverted pool of water that is caused by total internal reflection light.
- During a hot day air near the ground is warmer and therefore physically less dense than air away from the ground. Therefore on a hot day the refractive index increases gradually from the ground upwards.
- A ray of light travelling in air from sky to ground undergoes continuous refraction and finally reflected internally.



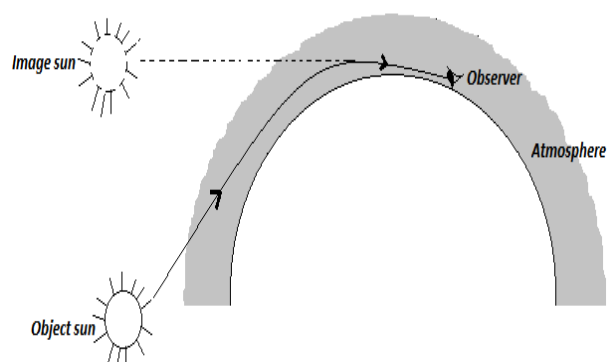
- Mirages are also witnessed in very cold regions in which the refractive index increases gradually from the ground upwards. Images appear inverted in the sky.

ii. Calculate the critical angle C



2. Atmospheric Refraction

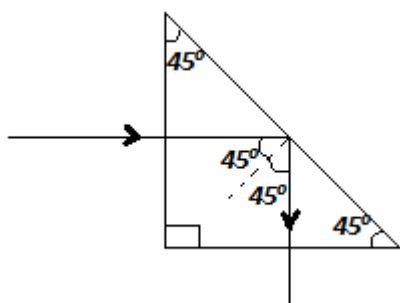
- This is a phenomenon in which light rays from the sun are refracted and then reflected internally towards the earth. As a result, the sun is seen even after it has set or before it rises.



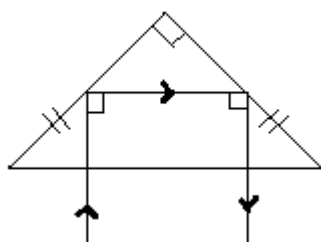
Total Internal Reflection Prisms

- Right-angled isosceles glass or Perspex prisms are used.

I. To turn a ray of light through 90°

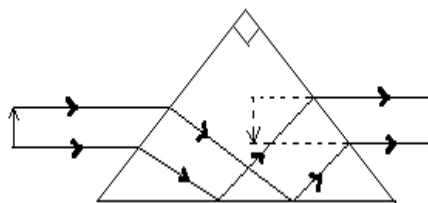


II. To turn a ray through 180°



III. Inversion with deviation

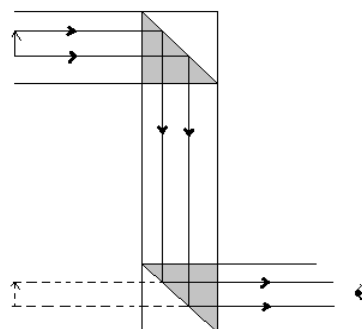
IV. Inversion without deviation



Applications of Total Internal Reflection

1. Periscope

- Light is deviated through 90° by first prism before the second prism deviates it further through 90° in the opposite direction. An upright virtual image is formed.

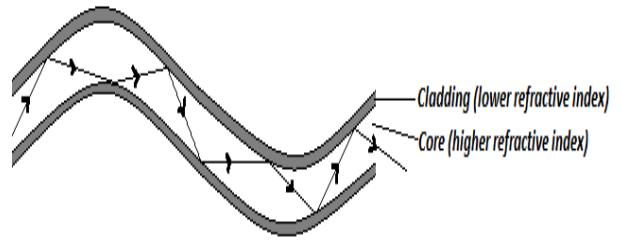
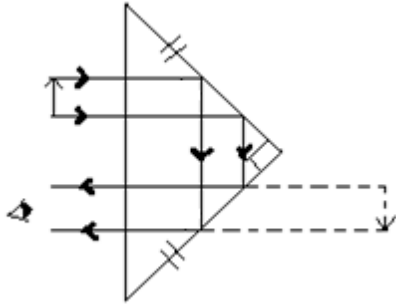


Note: Prisms are preferred to plane mirrors for use in periscopes and other optical instrument because:

- Mirrors absorb some of the incident light
- The silvering on mirrors can become tarnished and peel off
- Thick mirrors produce multiple images

2. Optical Fibre

- An optical fibre is a thin flexible glass rod of small diameter in the order of $10^{-6}m$. The central case of the glass is coated with glass of lower refractive index (**cladding**).



- A ray of light entering the fibre undergoes total internal reflections on the boundary of the high and low refractive index glass. The light therefore travels through the entire length of the fibre without any getting lost.

Advantages of optic fibers over ordinary cables.

-they have high carrying capacity

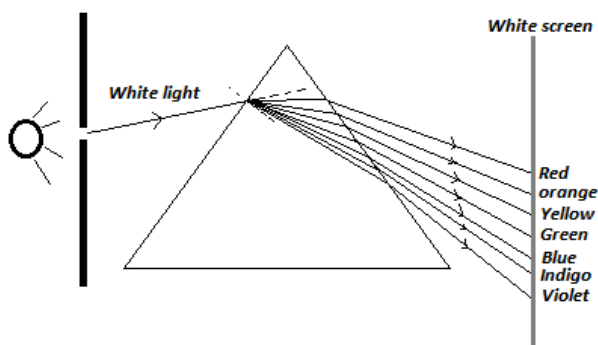
-they are thinner and lighter

Uses of Optical Fibre

- I. Used in medicine to view internal organs of the body.
- II. Used in telecommunication where they have higher advantage than ordinary cables since they have higher carrying capacity, they are thinner and lighter.

Dispersion of White Light

- Dispersion of light is the splitting of white light into its component colors. White light is a mixture of seven colors.
- The components of white light travel with same velocity in vacuum but their velocities are not the same in other media.



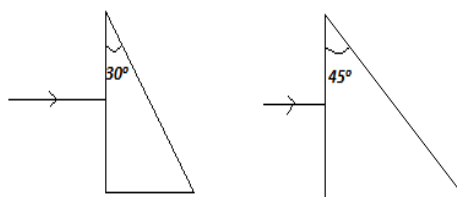
- **Cause of Dispersion of White Light:** The separation of white light into constituent colours is due to their different velocities in a given transparent medium.
- The **velocity of red is highest** while that of **violet is the least**. **Red colour has longest wavelength** while **violet the least wavelength** ($v = \lambda f$)

The Rainbow

- Rainbow is a bow-shaped colour band of the

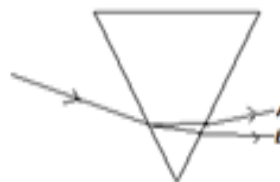
Revision Exercise

1. The diagram below show two prisms

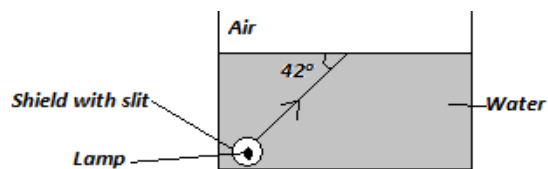


Given that the critical angle of the glass in both prisms is 42° sketch the paths of the two beams of monochromatic light until they leave the prisms.

2. The figure below show how white light behaves when it is incident on a glass prism.

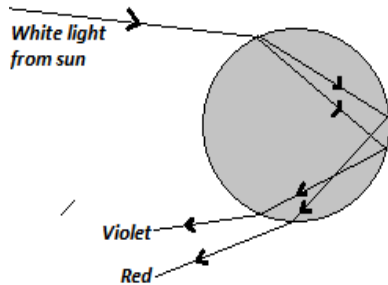


- I. Determine the critical angle of the glass material
 - II. Determine the refractive index of the glass material
3. The diagram below shows a transparent water tank containing water. An electric damp covered with a shield which has a narrow slit fixed at one near of the tank. A light ray from the slit reaches the water surface at an angle of 42° as shown below.

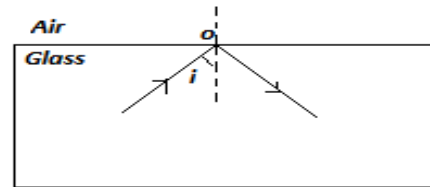


- I. Determine the angle of refraction for the ray shown in the diagram
 - II. Determine the angle of incidence for which the angle of refraction is 90° (refractive index of water = 1.33)
4. The diagram below shows a ray of light incident on the

visible spectrum seen in the sky. *It is formed when white light from the sun is **refracted**, **dispersed** and **totally internally** reflected by rain drops.*



glass – air interface from the inside of the glass. The angle of incidence i , is slightly smaller than the critical angle of glass.

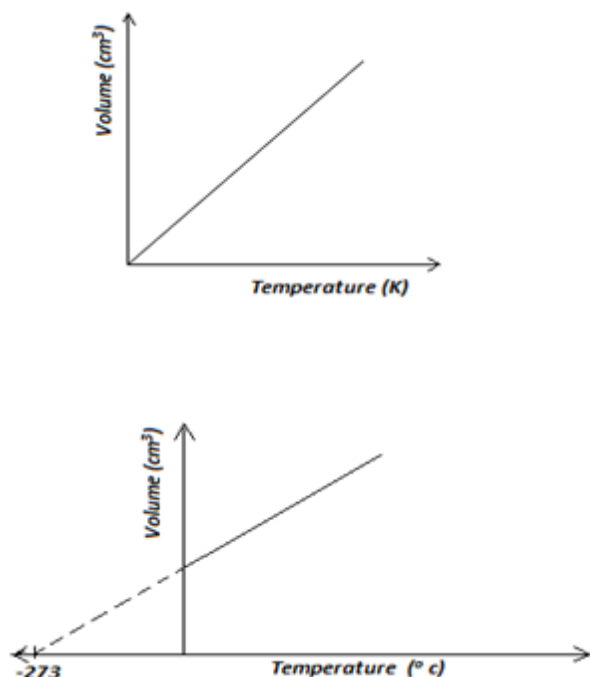


State and explain what would be observed on the ray if a drop of water was placed at the point of incidence, o

Specific objectives	Content
<p>By the end of this topic, the learner should be able to:</p> <ol style="list-style-type: none"> state the gas laws for an ideal gas verify experimentally the gas laws explain how the absolute zero temperature may be obtained from the pressure – temperature and volume-temperature graphs convert Celsius scale to kelvin scale of temperature state the basic assumption of the kinetic theory of gases explain the gas laws using the kinetic theory of gases solve numerical problems involving gas laws 	<ol style="list-style-type: none"> Boyle's law, Charles' law, pressure law, absolute zero Kelvin scale of temperature Gas laws and kinetic theory of gases Problems of gas laws (including $\frac{pv}{T} = \text{constant}$)
Introduction	Characteristics of ideal gas
<ul style="list-style-type: none"> <i>To study the behaviour of gases, pressure, volume and temperature of the gas are considered. A change in one of these variables causes the others to change. The gas laws deals with relationship between this parameters.</i> NB Gas laws apply for ideal gases only. An ideal gas is one that obeys the gas laws. 	<ol style="list-style-type: none"> Ideal gas contains identical particles of negligible volume There is no intermolecular forces of attraction between particles Molecules undergo perfectly elastic collision with other molecules and with the walls of the container.
Relationship between Kelvin (Absolute) and Celsius Scales of Temperature	The Gas Laws
<ul style="list-style-type: none"> Absolute zero temperature refers to the lowest temperature a gas can fall to. It <i>is -273° celcius (0 Kelvin)</i>. The kelvin scale of temperature starts at -273°c (0 K). The Kelvin (K) scale is related to the Celsius (°C) by: 	<p>Charles' law</p> <ul style="list-style-type: none"> It states that, <i>"the volume of a fixed mass of a gas is directly proportional to the absolute (kelvin) temperature at constant pressure"</i>. Mathematically, Charles' law can be expressed as:

<p>Temperature in kelvin, $T_K(K)$</p> <p>= Temperature in celcius, $T_C(^{\circ}C) + 273$</p> <p>Note: All temperatures must be expressed in kelvin any calculation.</p>	<p>$V \propto T$ (at constant pressure);</p> <p>$\Rightarrow V = kT$ wher k is a constant of proportionality</p> <p>$\frac{V}{T} = \text{constant}$</p> <p>Where V is the volume and T is the kelvin temperature</p> <p>Therefore, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$</p> <p>Where V_1, V_2 and T_1, T_2 are the initial and final values of volume and temperature respectively.</p>
<u>Exercise</u>	
<p>1. Convert the following temperature to kelvin</p> <p>I. $-40^{\circ}C$</p> <p>II. $55^{\circ}C$</p> <p>2. Convert the following values of temperature to degrees values</p> <p>I. $45K$</p> <p>II. $300K$</p>	

Graphically, Charles' law can be expressed as shown below;



Sample questions on Charles' law

1. State Charles' law for an ideal gas.

The volume of a fixed mass of a gas is directly proportional to the absolute (kelvin) temperature at constant pressure.

2. The set-up below shows an arrangement that can be used to determine the relationship between temperature and volume of a gas at constant pressure.

III. Describe how the set up can be used to verify Charles' law.

- The initial length of the air column is taken and recorded as well as the initial thermometer reading.
- The water bath is heated and new height (column) of air is taken and recorded with its corresponding temperature reading
- This is repeated several times at suitable temperature intervals to get several pairs of results
- A graph of volume (height, h (cm)) against absolute temperature is plotted.
- It is a straight line with positive gradient.
- This shows that the volume is directly proportional to absolute temperature.

3. The volume of a gas enclosed with a movable piston is 300 cm³ when the temperature is 20 °C. Determine the temperature at which the volume of the gas increases to 355 cm³. (Assume pressure does not change)

Solution

$$V_1 = 300 \text{ cm}^3, \quad V_2 = 355 \text{ cm}^3, \quad T_1 = 20^\circ\text{C} = (20 + 273)\text{K} = 293 \text{ K}, \quad T_2 = ?$$

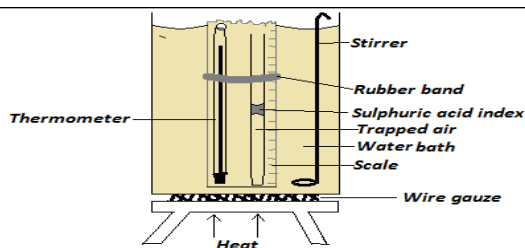
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{300 \text{ cm}^3}{293 \text{ K}} = \frac{355 \text{ cm}^3}{T_2}$$

$$T_2 = \frac{355 \text{ cm}^3 \times 293 \text{ K}}{300 \text{ cm}^3} = 346.72 \text{ K}$$

$$\text{In celsius scale, } T_2 = 346.72 - 293 = 53.72^\circ\text{C}$$

Exercise



I. State any two uses of sulphuric acid.

- Used as a pointer to volume of the gas on the scale
- Used as a drying agent for the air
- Used to trap air

II. What is the use of the stirrer?

To stir the water bath for uniform distribution of heat

III. State the measurement that used to be taken in this experiment.

- Temperature
- Air column height which corresponds to volume

1. In an experiment to find the relationship between the volume and temperature of a mass of air at constant pressure, the following results were obtained:

Volume (cm)	31	33	35	38	40	43
Temperature $^{\circ}\text{C}$	0	20	40	60	80	100

Draw a graph to show the relationship between volume and temperature and use the graph to calculate the increase in volume of the gas per unit rise in temperature

2. Some results of an experiment to study the effect of temperature on volume of a fixed mass of a gas at a constant pressure are displayed in the table below.

Volume (cm) ³	20	25			40
Temperature ($^{\circ}\text{C}$)	0		-136	205	

Fill the missing results

3. The volume of a gas enclosed with a movable piston is 0.02 m³ when the temperature is 42^oc. Determine the temperature at which the volume of the gas increases to 0.4 cm³ (assume pressure does not change)

4. A tube contains a gas enclosed with a thread of mercury. The tube was placed horizontally in a water bath. The initial temperature of the water was 20^oc and the length of the gas column was 25cm. Determine the temperature at which the length of air column would be 20cm (Assume the pressure does not change)

Pressure law

- Pressure law states that, ***“the pressure of a fixed mass of a gas is directly proportional to the absolute (kelvin) temperature at a constant volume”***.
- Mathematically**, pressure law can be expressed as:

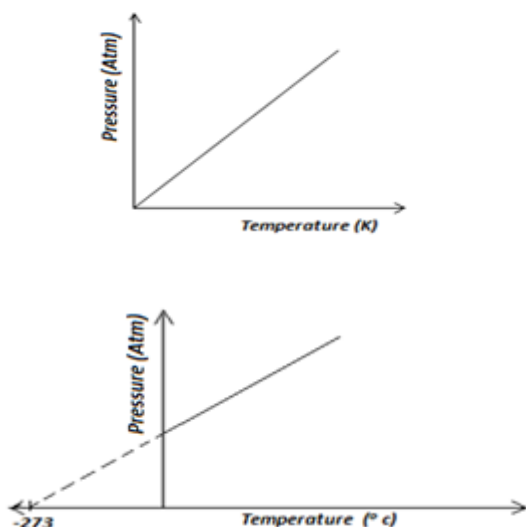
$$P \propto T(K), \text{ at constant temperature}$$

$$\frac{P}{T(K)} = \text{constant (at constant volume)}$$

$$\text{therefore, } \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Where P_1, P_2 and T_1, T_2 are initial and final pressure and temperature values respectively

- Graphically**, pressure law can be expressed as:



Sample questions on Pressure law

II. Describe how the set-up can be used to verify pressure law

- The initial temperature and pressure reading are taken and recorded
- The water bath is heated gently and some more pairs of pressure and temperature readings are taken and recorded at suitable temperature intervals
- A graph of pressure against temperature is plotted.
- It is a straight line with positive gradient.
- This shows that the pressure is directly proportional to absolute temperature.

3. A gas in container has pressure of 3.0×10^5 Pa when the temperature is 20°C . Determine the pressure of the gas when the temperature lowered to -5°C (assume there is no change in volume)

$$T_1 = 20^\circ\text{C} = 293 \text{ K}, \quad T_2 = -5^\circ\text{C} = 268 \text{ K}, \quad P_1 = 3.0 \times 10^5 \text{ Pa}$$

$$= ?$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{3.0 \times 10^5 \text{ Pa}}{293 \text{ K}} = \frac{P_2}{268 \text{ K}}$$

$$P_2 = \frac{3.0 \times 10^5 \text{ Pa} \times 268 \text{ K}}{293} = 2.744 \times 10^5 \text{ Pa}$$

Exercise

1. 80 cm^3 of hydrogen gas was collected at a temperature of 15°C and normal atmospheric pressure. Determine the pressure of a gas when the temperature is lowered to 0°C at constant volume.

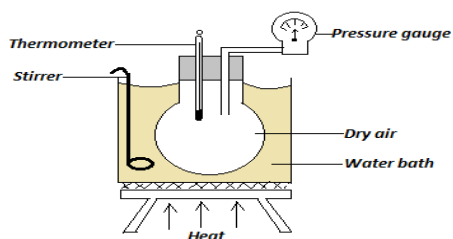
2. In an experiment to determine the absolute zero temperature (0 K), the pressure of a gas at constant volume was measured as the temperature was varied gradually. The table below shows the results obtained.

Pressure (mmHg)	75	77	80	82	85	88	90
	0	6	2	8	4	0	0

1. State pressure law.

The pressure of a fixed mass of a gas is directly proportional to the absolute (kelvin) temperature at a constant volume.

2. The set up below shows an arrangement used to determine the relationship between temperature and pressure of a gas at constant volume.



I. State the measurement that need to be taken in this experiment

a) Temperature

b) Pressure

- Graphically, Boyle's law can be expressed as:

							6
Temperature ^o C	15	25	35	45	55	65	75

- Plot a graph of pressure against temperature
- Determine the value of absolute zero from the graph
- Give a reason why the temperature in II above could not be practically obtained

Boyle's Law

- Boyle's law states that, **"the pressure of a fixed mass of a gas is inversely proportional to its volume at a constant temperature"**.
- Mathematically, Boyle's law can be expressed as:

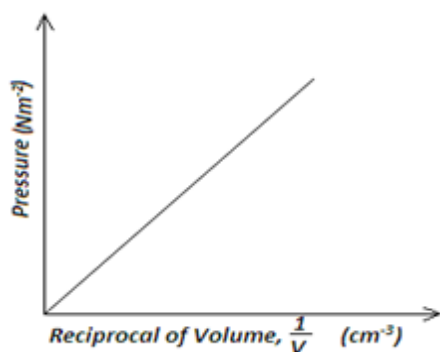
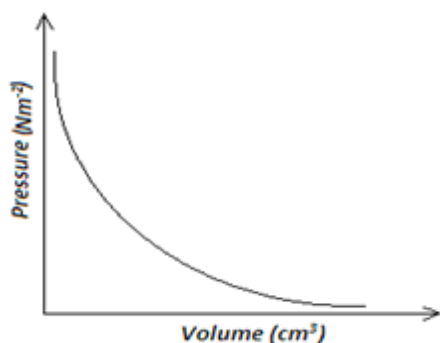
$$P \propto \frac{1}{V} \text{ (at constant temperature)}$$

$$PV = \text{constant (at constant temperature)}$$

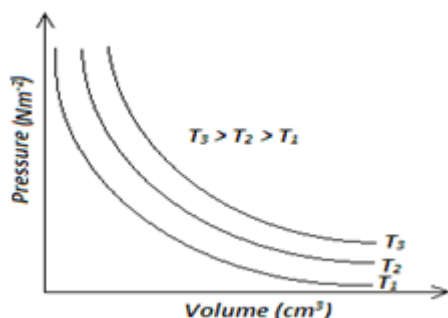
$$\text{therefore, } P_1 V_1 = P_2 V_2$$

where P_1, P_2 and V_1, V_2 are initial and final values of pressure and volume respectively.

Sample questions on Pressure law



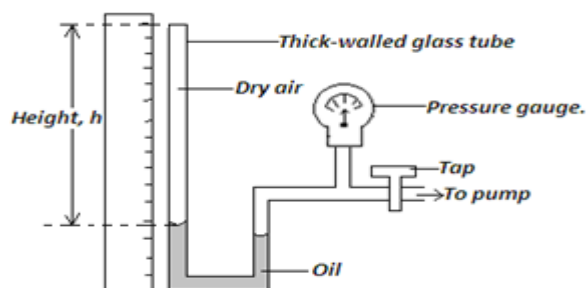
- If the experiment on Boyle's law is repeated at different fixed temperatures and the results plotted, isothermal curves are obtained.



1. State Boyle's law.

The pressure of a fixed mass of a gas is inversely proportional to its volume at a constant temperature.

2. The set-up below shows an arrangement used to determine the relationship between pressure and volume of a gas at constant temperature.



Describe how the set up can be used to verify Boyle's law.

- With the tap open, air is pumped until the oil raises a small but measurable height. The tap is then closed.
- The value of pressure and its corresponding height is read and recorded.
- This is repeated to obtain more pairs of values of pressure and corresponding heights.
- A graph of pressure versus volume is plotted using the result.
- It is a smooth curve with negative instantaneous gradient
- This shows that the pressure is inversely proportional to volume.

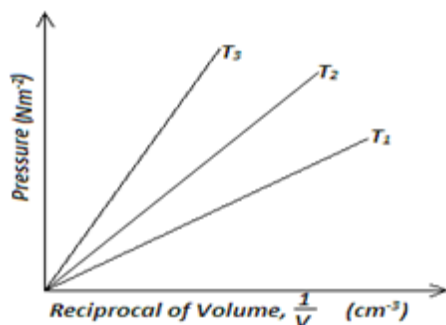
3. A bubble of air of volume 1.2cm^3 is released at the bottom of a water dam 15m deep. The temperature of the water is 20°C . Determine the volume of the bubble as it emerges on the water surface where the pressure is the normal atmospheric pressure (take ρ of water $=1000\text{kgm}^{-3}$ and normal atmospheric pressure $1.0 \times 10^5\text{pa}$)

Solution

$$P_1 V_1 = P_2 V_2$$

$$P_1 = P_{\text{atm}} + \rho gh$$

$$1.0 \times 10^5\text{pa} + 1000\text{kgm}^{-3} \times 10\text{Nkg}^{-1} \times 15\text{m} = 2.5 \times 10^5\text{pa}$$



$$P_2 = 1.0 \times 10^5 \text{ pa}$$

$$V_1 = 1.2 \text{ cm}^3$$

$$V_2 = ?, V_2 = \frac{P_1 V_1}{P_2}$$

$$= \frac{2.5 \times 10^5 \text{ pa} \times 1.2 \text{ cm}^3}{1.0 \times 10^5 \text{ pa}} = 3.0 \text{ cm}^3$$

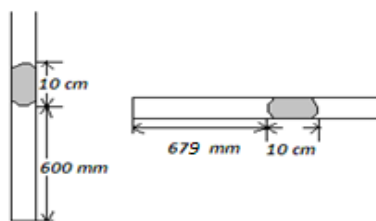
3. A narrow uniform glass tube contains air enclosed by a 10cm thread of mercury. When the tube is held vertical, the air column is 600 mm long. When tilted slightly, the air column is 679 mm long. The temperature is the same in both cases (i.e. temperature is constant)

1. Give the reason for the differences in length of the air column for the two positions.

When vertical, the mercury exerts pressure on the air, reducing the volume.

II. Determine the atmospheric pressure at the place (ρ of mercury = 13.6 g cm^{-3})

Solution



$$P_1 V_1 = P_2 V_2$$

$$P_1 = P_{\text{atm}} + \rho gh; \quad P_1 = P_{\text{atm}} + 13600 \text{ kg m}^{-3} \times 10 \text{ N kg}^{-1} \times 0.1 \text{ m}$$

$$= P_{\text{atm}} + 1.36 \times 10^4 \text{ pa}$$

$$P_2 = P_{\text{atm}}$$

Let the cross-sectional area of the tube be A

$$V_1 = 60A \text{ cm}^3, \quad V_2 = 67.9A \text{ cm}^3$$

$$(P_{\text{atm}} + 1.36 \times 10^4 \text{ pa}) \times 60A \text{ cm}^3 = P_{\text{atm}} \times 67.9A \text{ cm}^3$$

$$P_{\text{atm}} = \frac{8.16 \times 10^5 \text{ pa}}{7.9} = 1.03 \times 10^5 \text{ pa}$$

Exercise

1. A column of air 26 cm long is trapped by mercury thread 5 cm long when vertical. When it is placed horizontally, the air column is 28 cm. Find the atmospheric pressure in mmHg.

2. The table below shows the results obtained in an experiment to study the variation of the volume of a fixed mass with pressure at constant temperature.

Pressure (cm Hg)	60	-	90	-
Volume (cm^3)	36	80	-	40

Fill in the missing results.

3. Explain why a bubble increases in size and finally burst when it reaches the surface.

$$\frac{P_1 V_1}{T_1(K)} = \frac{P_2 V_2}{T_2(K)}$$

$$\frac{1.0 \times 10^5 \text{ Pa} \times 1 \text{ m}^3}{100 \text{ K}} = \frac{P_2 \times 0.06 \text{ m}^3}{293 \text{ K}}$$

$$P_2 = \frac{1.0 \times 10^5 \text{ Pa} \times 1 \text{ m}^3 \times 293 \text{ K}}{100 \text{ K} \times 0.06 \text{ m}^3} = 4.883 \times 10^6 \text{ Pa}$$

Exercise

1. A container carries 3000 cm^3 of oxygen at a pressure of $1.0 \times 10^6 \text{ pa}$ a temperature of 20°C in a cylinder. What is the volume of the gas in the cylinder at the top of the mountain where pressure is $0.8 \times 10^6 \text{ pa}$ and temperature is -17°C ?

Kinetic Theory of Gases

- The kinetic theory of gases proposes that the molecules of a gas are in a continuous random motion.

Basic Assumptions of the Kinetic Theory of Gases

- I. Attraction between the molecules of a gas is negligible.
- II. The volume of the molecule of the gas is zero.
- III. Collisions between the molecules and with the walls of the container and perfectly elastic.

Kinetic Theory of Gases and Gas Laws

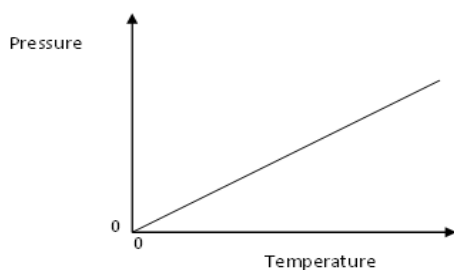
Boyle's Law and Kinetic Theory

- When temperature is constant, a change in volume of gas results in a change in number of collisions per unit time between molecules and between molecules and walls of container. As a result, pressure changes.

Charles' Law and Kinetic Theory

<p>General Gas Equation (Equation of State) for Ideal Gases</p> <ul style="list-style-type: none"> It is obtained by combining any two of the three gas law. $\frac{PV}{T(K)} = \text{constant}$ <p>Therefore, $\frac{P_1 V_1}{T_1(K)} = \frac{P_2 V_2}{T_2(K)}$</p> <p>where P_1, P_2, V_1, V_2 and T_1, T_2 are the initial and final values of pressure, volume and temperature respectively.</p>	<ul style="list-style-type: none"> When pressure is constant and temperature is raised, the speed of molecules rises causing them to occupy a larger volume of the container. <p>Pressure Law and Kinetic Theory</p> <ul style="list-style-type: none"> A change in temperature changes the kinetic energy and hence the speed of molecules of a gas and therefore if volume of gas is constant, pressure changes as a result change in temperature. Pressure is caused by collisions between molecules of the gas and with the walls of the container. <p>Limitation of Gas Laws</p> <ul style="list-style-type: none"> Gas laws do not apply in real gases. Real gases liquefy before the volume of the gas reduces to zero.
<p style="text-align: center;">Example</p> <p><i>In the manufacture of oxygen 1m^3 of the gas produced at -173°C and normal atmospheric pressure is compressed into a cylinder of volume 0.06 m^3 and stored at a temperature of 20°C. What is the pressure of the gas in the cylinder? (Normal atmospheric pressure, $P_{\text{atm}} = 1.0 \times 10^5\text{ pa}$)</i></p> <p><u>Solution</u></p> <p>$P_1 = 1.0 \times 10^5\text{Pa}, \quad P_2 = ? \quad , \quad V_1 = 1\text{ m}^3, \quad V_2 = 0.06\text{ m}^3,$</p> <p>$T_1 = -173^\circ\text{C} = 100\text{ K}, \quad T_2 = 20^\circ\text{C} = 293\text{ K},$</p>	
<p style="text-align: center;">Revision Exercise</p>	

1. A balloon filled with organ gas a volume of 200 cm^3 at the earth's surface where the temperature is 20°C , and the pressure 760mm of mercury. If it is allowed to ascend to a height where the temperature is 0°C and the pressure 100mm of mercury, calculate the volume of the balloon.
2. A fixed mass of Oxygen occupies a volume of 0.01 m^3 at a pressure of $1 \times 10^5 \text{ Pa}$ and a temperature 0°C . If the pressure is increased to $5 \times 10^5 \text{ pa}$ and the temperature is increased to 25°C . What volume will the gas occupy?
3. An empty barometer tube of length 90cm is lowered vertically with its mouth downwards into a tank of water. What will be the depth at the top of the tube when the water has risen 15cm inside the tube, given that the atmospheric pressure is 10 m of water?
4. A hand pump suitable for inflating a football has a cylinder which is 0.24m in length and an internal cross-sectional area of $5.0 \times 10^{-4} \text{ m}^2$. To inflate the football the pump handle is pushed in and air is pumped through a one-way valve. The valve opens to let air in to the ball when the air pressure in the pump has reached 150 000 pa. (Assume the air temperature remains constant)
 - I. If the pressure in the pump is initially 100 000 pa, calculate how far the piston must be pushed inwards before the one way valves opens.
 - II. When the one-way valve opens the total pressure in the cylinder will be 150 000 pa. What force will be exerted on the piston by the air in the cylinder?
5. The graph in figure below shows the relationship between the pressure and temperature for an ideal gas. Use this information in the figure to answer questions that follow.



- I. State the unit of the horizontal axis

quantity.

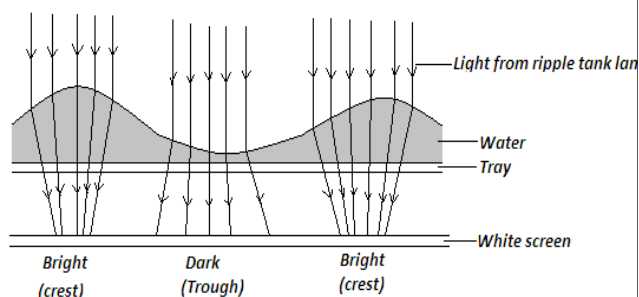
II. Write a statement of the gas law represented by the relationship.

- 6. Draw axes and sketch a graph of pressure (p) against reciprocal of volume ($1/v$) for a fixed mass of an ideal gas at a constant temperature.**
- 7. A balloon is filled with air to a volume of 200ml at a temperature of 293 k. Determine the volume when the temperature rises to 353 k at the same pressure**
- 8. Show that the density of a fixed mass of gas is directly proportional to the pressure at constant temperature.**
- 9. The pressure of helium gas of volume 10cm^3 decreases to one third of its original value at a constant temperature. Determine the final volume of the gas.**

<u>Specific objectives</u>	<u>Content</u>
<p>By the end of this topic the learners should be able to:</p> <ul style="list-style-type: none"> a) Describe experiments to illustrate properties of waves b) Explain constructive and destructive interference c) Describe experiments to illustrate stationary waves 	<ul style="list-style-type: none"> 1. Properties of waves including sound waves: Refraction, diffraction, interference (experimental treatment required) 2. Constructive interference and destructive interference (Qualitative treatment only) 3. Stationary waves (qualitative and experimental treatment required)

The Ripple Tank

- A **ripple tank** is used to demonstrate properties of waves in the laboratory. It consists of a **tray** containing water, a **point source of light** above the tray, a **white screen** placed underneath and a **small electric motor (vibrator)**.
- The waves are generated by an electric vibrator as ripples and they travel across the surface of the shallow water in the tray. **Crests appear bright** while **troughs appear dark** when the wave is illuminated. Note that crests act as plano-convex lens and therefore converge rays on them while troughs act as plano-concave and therefore diverge ray incident on them.

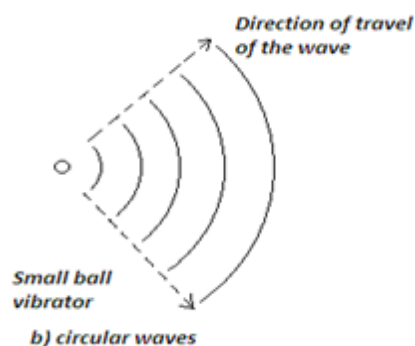
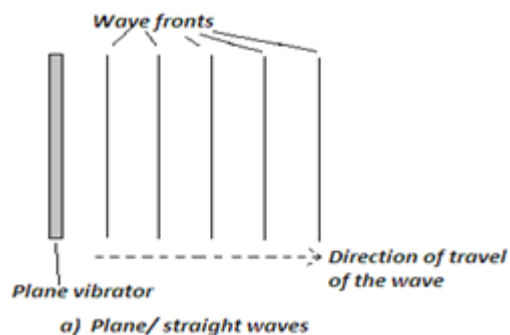


- A **bar** attached to vibrator produces **plane waves** while **circular waves** are produced by **fixing a small ball to the bar**.

Properties of Waves

1. Rectilinear Propagation

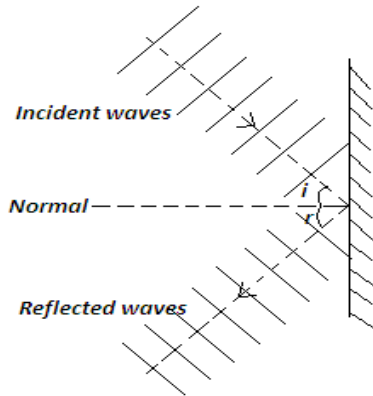
- Rectilinear propagation is the property of the waves to travel in a straight line and perpendicular to the wave fronts.



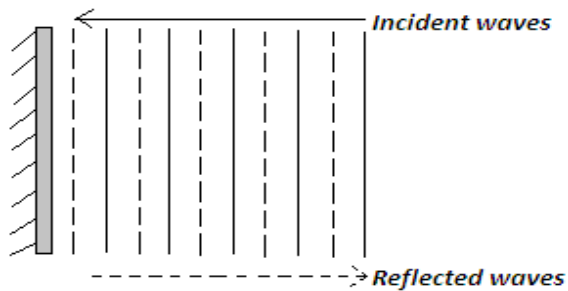
2. Reflection of Waves

- Reflection is the bouncing off of a wave from an obstacle. When reflection takes place, only the **direction of the wave changes**, **Wavelength**, **frequency** and **speed** of the **wave do not change**. Reflection of waves obeys the laws of reflection which are applicable to the reflection of light.

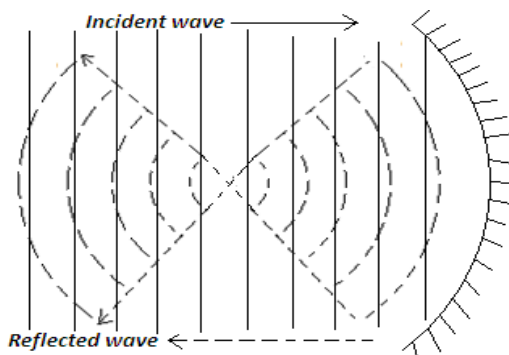
a) Plane waves on a straight reflector at an angle



b) Plane waves on a straight reflector at an angle of 90°



c) Plane waves on a concave reflector

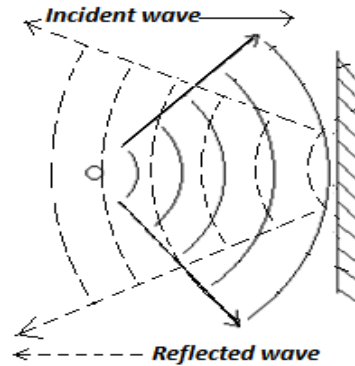


After reflection, the waves converge to a point in front of the reflecting surface. Hence, the concave reflector has a **real focus**

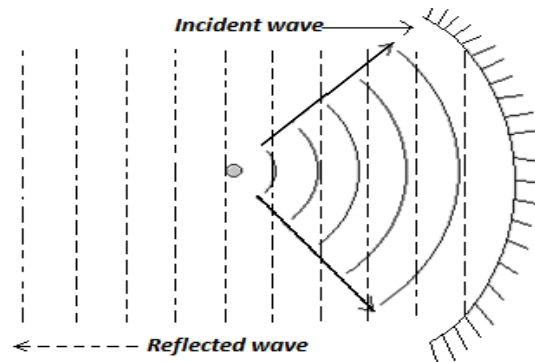
d) Plane waves on a convex reflector

After reflection, the waves appear to be diverging from a point behind the mirror convex reflector has a **virtual focus**.

e) Circular waves on a straight reflector

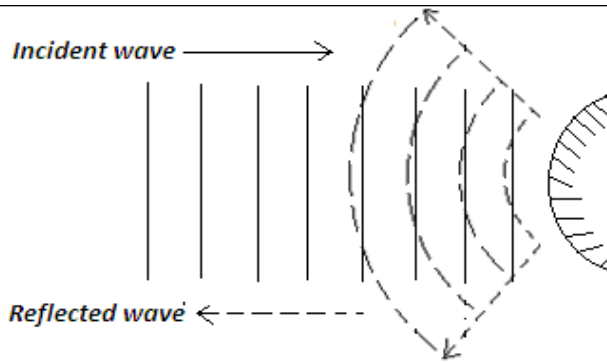


f) Circular waves on a concave reflector

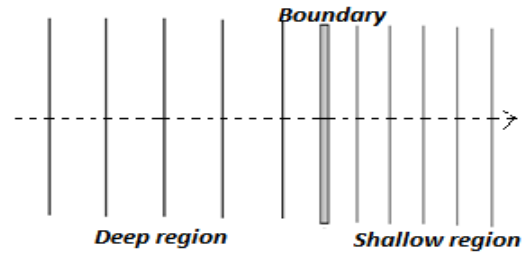


3. Refraction of Waves

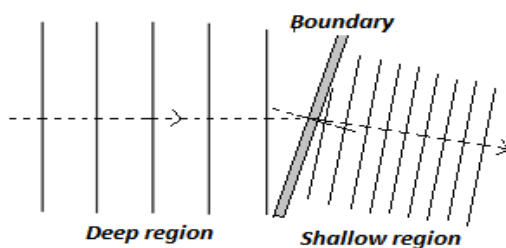
- Refraction of a wave refers to change in direction of the wave. When a wave is refracted, **it changes its speed, direction and wavelength but not its frequency**.
- When water waves cross into shallow region from a deep region, the separation between wave fronts becomes smaller i.e. the wavelength decreases



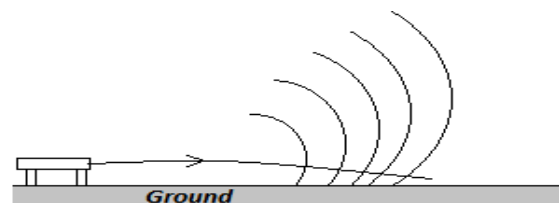
a) *Wave fronts parallel to boundary*



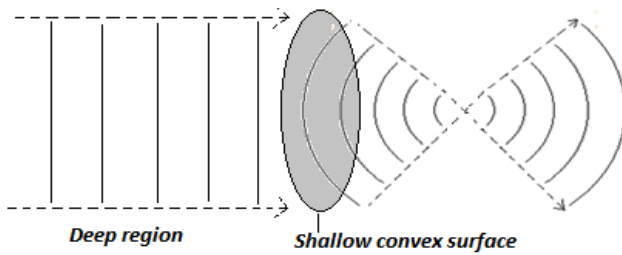
b) *Wave fronts meeting the boundary at an angle*



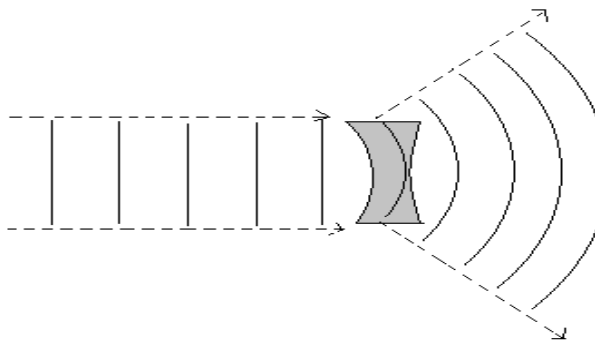
- *During the night* air close to ground is cooler than that higher above. Sound waves produced close to ground are refracted downwards because the wave fronts near the ground move slower than those on the upper parts. ***This is why sound waves travel far from source during the night.***



c) *Refraction of straight water waves on shallow convex surface*

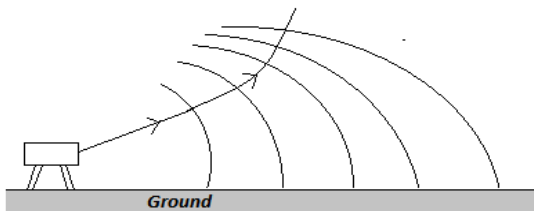


d) *Refraction of straight water waves on shallow concave surface*



Refraction of Sound Waves

- **During the day** air close to ground is much warmer than the air higher above. Sound waves produced close to the ground are refracted upwards because wave fronts near the ground move faster than those on the upper parts. **This is why sound waves are not heard far from source during the day.**



Exercise

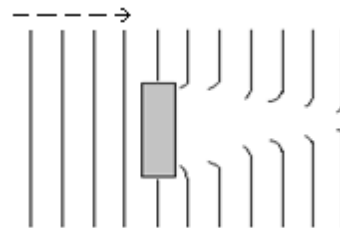
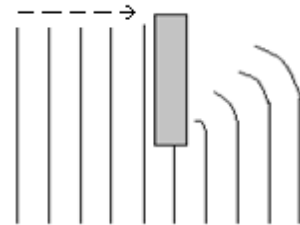
The figure below shows plane waves travelling in a shallow region of a ripple tank. The shallow region is incident on a deeper region at an angle of 45° as shown.



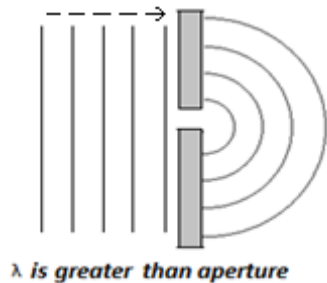
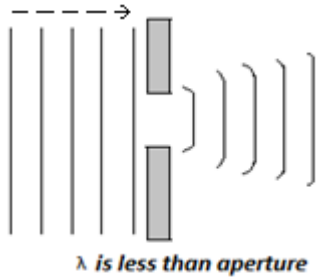
- Describe how the waves are generated in a ripple tank
- Complete the diagram to show the appearance of the wave fronts in the deep region
- What property of waves is illustrated in the diagram you have drawn?

4. Diffraction of Waves

- Diffraction is the spreading of a wave as it goes around an obstacle or through a small aperture. When diffraction occurs there is a **change in direction** but **not in velocity, frequency or wavelength**.



- When aperture is wider than the wavelength of the plane wave, the wave passes through as plane waves but when the width of the aperture is nearly equal or less than the wavelength of the plane wave, the wave fronts emerge circular



- Diffraction of sound waves is the reason as to why sound from a loud speaker in a room is heard round a corner without the source being seen.

Exercise

5. Interference of Waves

- Interference is the interaction of two or more waves of the same frequency emitted from coherent sources. The result of interference can be a much longer wave, a smaller wave or no wave at all.
- Interference is an import of the principle of superposition which states that: ***"the resultant effect of two waves travelling at a given point in the same medium is the vector sum of their respective displacement"***

Condition for interference

The wave sources ***must be coherent*** i.e.

- I. Have same frequency or wavelength
- II. Have equal or comparable amplitudes
- III. Have constant phase difference

Types of interference

a) Constructive interference

- It occurs when the wave amplitudes reinforce each other building a wave of even greater amplitude

1. Explain why diffraction of light waves is not a common phenomenon.

2. Give the definition of the following terms as connected with waves

- a) Wavelength
- b) Frequency
- c) Wave front

3. Five successive wave fronts in a ripple tank are observed to spread over a distance of 6.4cm. If the vibrator has a frequency of 8Hz, determine the speed of the waves

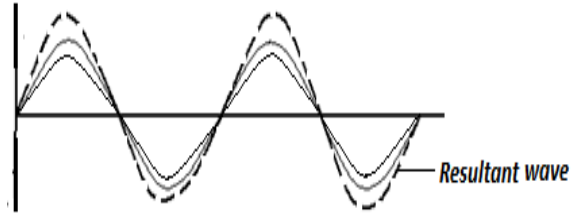
4.

- a) What is diffraction?
- b) What factors determine the extent of diffraction that occurs?
- c) Describe an experiment that can be set to illustrate this phenomenon

5. Diffraction, refraction and reflection are all properties of waves which one of these affects:

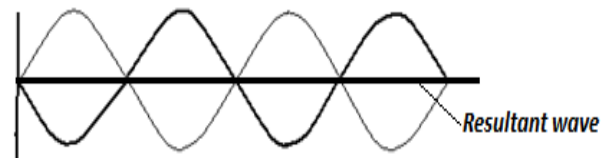
- a) Direction but not speed?

Speed and direction of travel of the waves?



b) Destructive interference

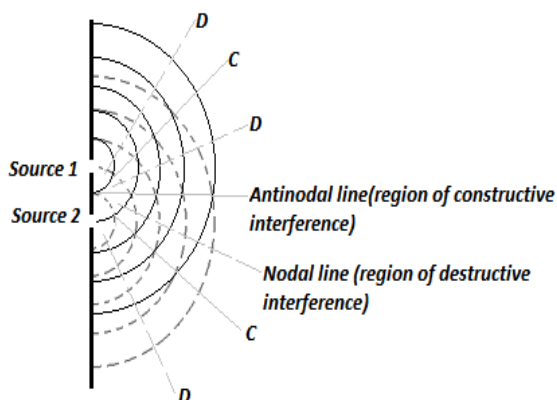
It occurs when the wave amplitude oppose each other resulting in waves of reduced amplitude.



- For this case, the waves undergo complete destructive resulting in a pulse (a wave) of zero amplitude

Interference in water (using a ripple tank)

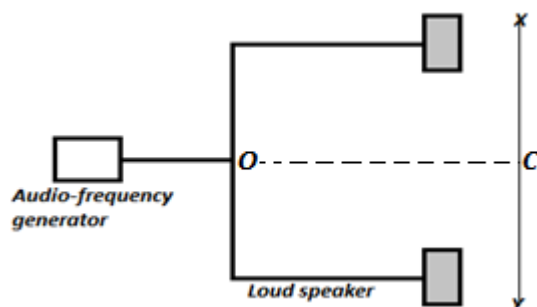
- **Coherent water waves are generated** by attaching two similar balls on the bar in contact with vibrator.



- **Observation:** Alternate dark lines (in regions of destructive interference) and bright lines (in regions of constructive interference) are seen on the white screen.

Interference in sound

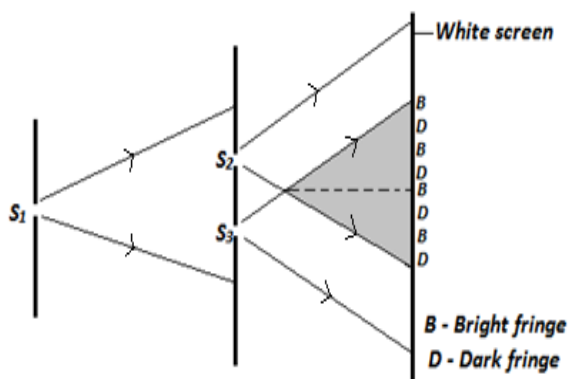
- **Coherent sources of sound** can be obtained by connecting two identical loud speakers in parallel to an audio-frequency generator as show below.



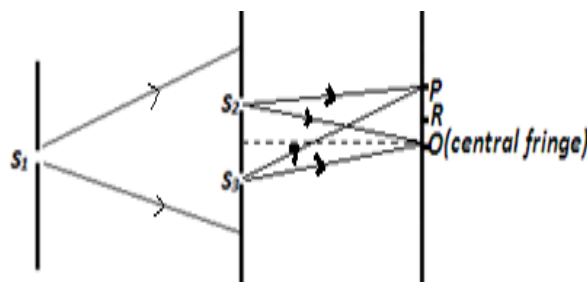
- Alternate loud (in regions of constructive

Interference in light (Young's double slit experience)

- Interference in light is evidence that light is a wave. It can be demonstrated by young's double slit experiment whose pioneer is the **Physicist Thomas Young**.



- A monochromatic light source is used in the double slit experiment. The slit S_1 diffracts light falling on it illuminating both slit S_2 and S_3 in front of it.
- A series of alternate bright (in regions of constructive interference) and dark (in regions of destructive interference) vertical bands/fringes are formed on the screen.
- The central fringe is the brightest (region of constructive interference).



$$S_2O = S_3O \text{ implying that path difference is zero}$$

$$S_3P - S_2P = 1\lambda \text{ i.e. path difference one wave.}$$

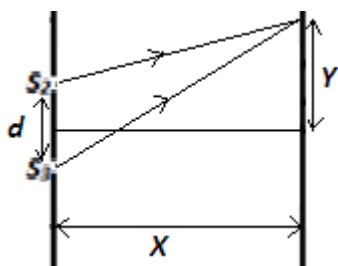
$$S_3R - S_1R = \frac{1}{2}\lambda \text{ i.e. path difference} = \frac{1}{2}\text{wave}$$

interference) and soft sound (in regions of destructive interference) is heard along XY.

- Along CO only loud sound is heard
- If waves from one speaker are exactly out of phase with those from the other soft sound will be heard along CO i.e. destructive interference
- Connecting the speakers to same audio frequency generator makes them satisfy the condition of being coherent sources.
- If the frequency of the signal is increased, the points of constructive interference (loud sounds) along XY will become more closely spaced and same way to those of destructive interference

Complete the diagrams to show the pattern across the slits.

III. The figure below shows two rays of monochromatic light incident on two adjacent slits S_2 and S_3 .



Give an expression for the wave length of the light in terms of d , x , and y

IV. In the space below, sketch the interference pattern observed if white light was used instead of monochromatic light

V. Explain the variation of frequency across the pattern displayed in (IV) above

VI. Give that the wavelength of the monochromatic light used in (III) above is 1.0×10^7 m, calculate its frequency

Example

1. I. Distinguish between diffraction and refraction of waves

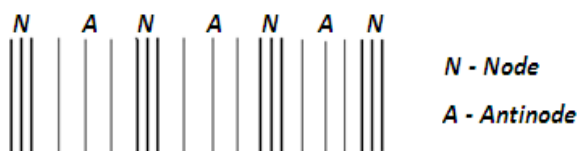
II. The figures below shows plane waves approaching slits



- A **node** is a point of zero displacement of a stationary wave while **an antinode** is a point of maximum displacement of a stationary wave.

2. Stationary longitudinal wave

- An example is a slinky spring with one end fixed and the other end moved to and fro rapidly in a horizontal direction.



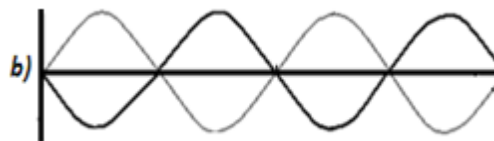
- Another example is when two speakers connected to same audio- frequency generator are arranged to face each other.



(speed of light is $3.0 \times 10^8 \text{ ms}^{-1}$)

2. Two observers P and Q are stationed 2.5km apart, each equipped with a starter gun. Q fires the gun and observes P record the sound 7.75 seconds after seeing the smoke from the gun. Later P fires the gun and observes Q record sound 7.25 seconds after seeing the smoke from the gun. Determine;

- The speed of sound in air
- The component of the speed of the wind along the straight line joining P and Q



- Alternate loud (at antinode) and soft (at node) is heard along AB.
- Diagram b) shows the wave form formed on the screen of a CRO when a microphone connected to the CRO is moved along AB.

Stationary(Standing) Waves

- It is defined as a wave formed when two equal progressive waves travelling in opposite directions are superposed on each other.

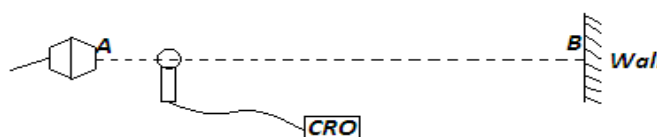
Types of stationary waves

1. Stationary transverse wave

- An example this case is a wave produced when jerking a string fixed at one end. A transverse wave travels along the string to the fixed end and then reflected back. The two waves travelling in opposite directions along the string then combine/ superpose to form a stationary transverse wave.

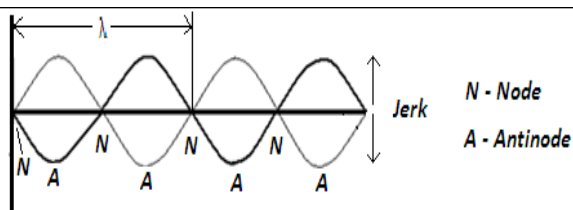
Exercise

The diagram below shows an arrangement that can be used to determine the speed of sound in air.



A microphone connected to a CRO with its time base on is moved along an imaginary line AB between the wall and the loud speaker.

- Use a diagram to explain what is observed as the microphone is moved from A to B
- If the frequency of the sound emitted by the loud speaker is 1650 Hz and the distance between a minimum and the next maximum is 0.05m, calculate the velocity of sound in air.



- III. If the frequency of the vibrating loud speaker is decreased what happens to the distance between two adjacent maximum?

Conditions Necessary for Formation of Stationary Waves

- The two progressive waves travelling in opposite directions to form a stationary wave **must be:**

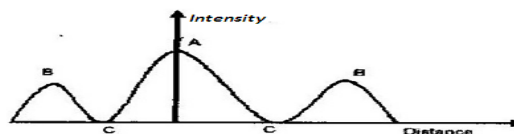
- Same speed
- Same frequency
- Same or nearly equal amplitudes

Properties of a Stationary Wave

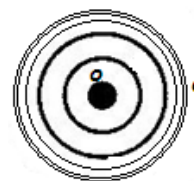
- A stationary wave is produced by superposition of two progressive waves travelling in opposite directions.
- The wave has nodes at points of zero displacement and antinodes at points of maximum displacement.
- In the wave, vibration of particles at points between successive nodes is in phase.
- Between successive nodes particles have different amplitudes of vibration.
- The distance between successive nodes or antinodes is $\frac{\lambda}{2}$.

Differences between stationary and progressive

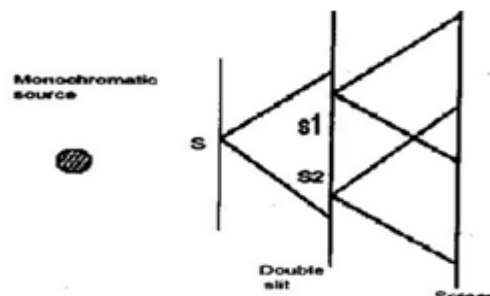
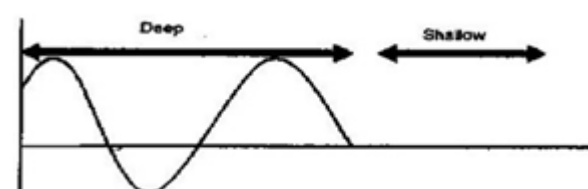
5. The sketch graph shows the results of an experiment to study diffraction patterns using double slit.



- Sketch an experimental set up that may be used to obtain such a pattern.
 - Name an instrument for measuring intensity
 - Explain how the peaks labelled A and B and troughs labeled C are formed.
6. What measurable quantity is associated with colours of light?
7. Circular water waves generated by a point source at the centre O of the pond are observed to have the pattern shown in the Fig. Explain the pattern.



8. In an experiment to observe interference of light waves, a double slit is placed close to the source.
- State the function of the double slit.
 - Describe what is observed on the screen.

waves		iii. State what is observed on the screen when
Stationary waves	Progressive waves	a) The slit separation S_1S_2 is reduced.
They do not transfer energy from one point to another since the wave forms do not move through the medium	They transfer energy from one point to another since the wave forms move through the medium	b) White light is used instead of monochromatic source.
The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$	Distance between successive troughs or crests is λ	13. The Fig. shows an experimental arrangement. S_1 and S_2 are narrow slits.
Vibrations of particles at points between successive nodes are in phase	Phase of particles near each other are different	
The amplitudes of particles between successive nodes is different	The amplitude of any two particles which are in phase are the same	State what is observed on the screen when the source is:-
		i) Monochromatic (ii) White light
		14. The fig shows the displacement of a particle in progressive wave incident on a boundary between deep and shallow regions.
		
Revision Exercise		I. Complete the diagram to show what is observed after boundary. (Assume no loss of energy).
1. Name a property of light that shows it is a transverse wave.		II. Explain the observation in (i) above.
2. In an experiment using a ripple tank the frequency, f of the electric pulse generator was reduced to one third of its original value. How does the new wave length compare with the initial wavelength? Explain your answer.		
3. Distinguish between stationary and progressive waves.		
4. State the condition for a minimum to occur in an interference pattern.		
Chapter Seven		CURRENT ELECTRICITY II

Objectives	Content
<p>By the end of this topic the learner, should be able to:</p> <ol style="list-style-type: none"> Define potential difference and state its units Measure potential difference and current in a circuit Verify Ohm's law Define resistance and state its unit Determine experimentally the voltage-current relationships for various conductors Define emf and explain internal resistance of the cell Derive the formulae for effective resistance of resistors in series and in parallel Solve numerical problems involving ohm's law resistor in the series and in parallel 	<ol style="list-style-type: none"> Scale reading ammeter, voltmeter Electric circuits: current, potential difference Ohm's law (experimental treatment required) Resistance types of resistors, measurements of resistance units Electromotive force(emf) and internal resistance of a cell($E = V + Ir$) Resistors in series and in parallel Problems on ohm's law, resistors in series and in parallel.
Electric Current	
<ul style="list-style-type: none"> Electric current refers to the rate of flow of charge. The movement of charged particles called electrons constitutes an electric charge and the conducting path through which electrons move is called an electric circuit. $\text{current, } I = \frac{\text{change, } Q}{\text{time, } t}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $I = \frac{Q}{t}$ </div> <ul style="list-style-type: none"> SI unit of electric current is the ampere (A) after the famous physicist Marie Ampere. Sub-multiples of the ampere are milli- 	$I = \frac{ne}{t} \quad n = \frac{It}{e}$ <p>NB charge on an electron, $e = 1.6 \times 10^{-19}$ coulomb</p> <div style="background-color: #f0f0f0; padding: 5px; text-align: center;">Example</div> <p>Calculate the amount of charge that passes through a point in a circuit in 3seconds, if the current in the circuit is 0.5A.</p> <div style="background-color: #f0f0f0; padding: 5px; text-align: center;">Solution</div> <p>$Q = It$</p> <p>$Q = 0.5 \text{ A} \times 3\text{s} = 1.5 \text{ C}$</p> <div style="background-color: #f0f0f0; padding: 5px; text-align: center;">Exercise</div>

ampere(mA) and micro-amperes (μA)

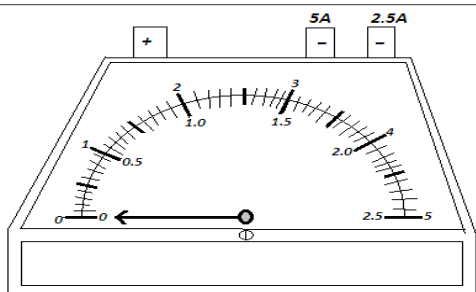
$$1 \text{ mA} = 1 \times 10^{-3} \text{ A}, 1 \mu\text{A} = 1 \times 10^{-6} \text{ A}$$

Definition of the ampere

- **An ampere** refers to an electric current that flows in a conductor when a charge of 1 coulomb flows per unit time

Total charge passing through a point in a circuit

- If n electrons pass through a point and that each electron carries a charge e then the total charge Q passing through the point is equal to ne coulombs i.e. $Q = ne$; but $Q = It$, $\Rightarrow ne = It$



1. A current of 0.08A passes in a circuit for 2.5 minutes.

I. How much charge passes through a point in the circuit?

II. Calculate the number of electrons passing through the point per second

2. A current of 0.5A flows in a circuit. Determine the quantity of charge that crosses a point in 4 minutes.

Measurement of Electric Current

- Electric current is measured using **an ammeter**.

Example

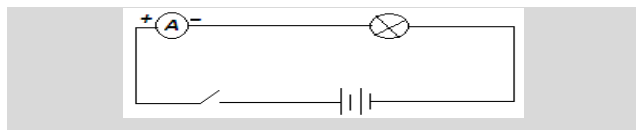
In moving a charge of 30 coulombs from point B to A 150 joules of work done what is the decimal place between A and B?

Solution

$$\text{p.d} = \frac{W}{Q}$$

$$\text{p.d} = \frac{150}{30} = 5 \text{ V}$$

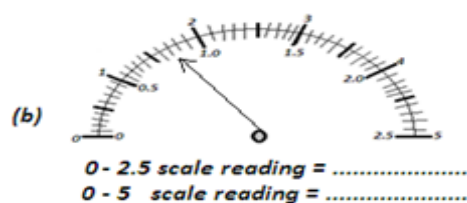
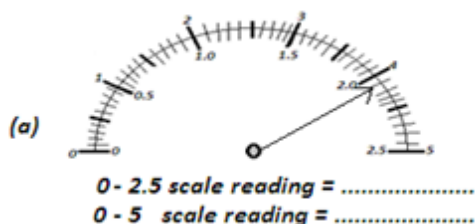
- **Zero error**, if any, should be rectified using the adjustment screw before using the ammeter.
- The ammeter is **connected in series** with other components in the circuit since **it is an instrument of low resistance**.



- **An appropriate scale** should be chosen to safe guard the coil of the ammeter from blowing up
- **Accuracy** of each scale of the ammeter must be observed when recording readings.

Exercise

Give the readings shown by both scales of the ammeter below.

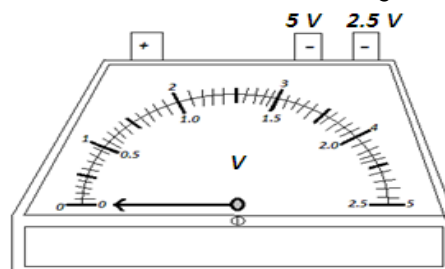


Definition of the volt

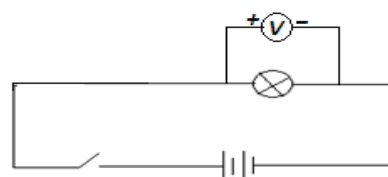
- A volt is defined as the energy needed to move one coulomb of charge from one point to another

Measurement of Potential Difference

- Potential difference is measured using a **voltmeter**.



- **Zero error**, if any, should be rectified using the adjustment screw before using the voltmeter.
- The voltmeter is **connected in across (in parallel)** the components in the circuit since **it is an instrument of high resistance**.



- **An appropriate scale** should be chosen to safe guard the coil of the voltmeter from blowing up.
- **Accuracy** of each scale of the voltmeter must be observed when recording readings.

Exercise

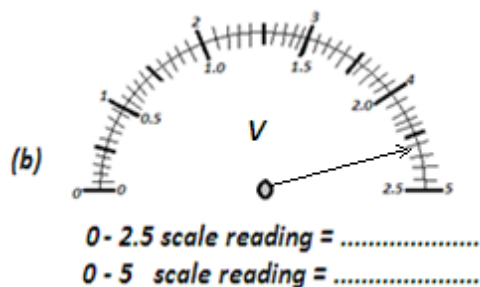
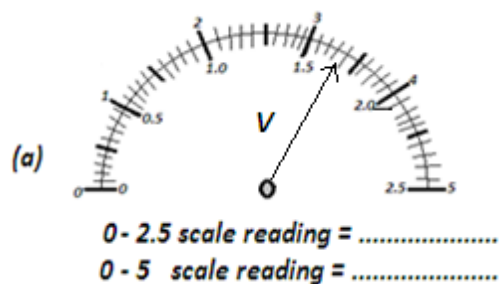
Give the readings shown by both scales of the voltmeter below.

Potential Difference (Voltage)

- *Electric potential difference between two points refers to work done (in joules) in moving one coulomb of charge from one point to the other.*
- The **SI unit** of potential difference is **the volt (V)**.
- **The battery** is the source of power for moving charge through the circuit.

$$\text{potential difference} = \frac{\text{work done, } W(\text{in joules})}{\text{Change moved, } Q(\text{in coulombs})}$$

$$\text{p.d} = \frac{W}{Q}$$

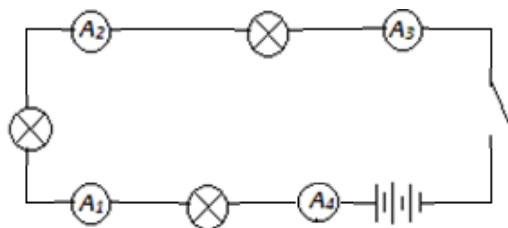


Note: For both the ammeter and voltmeter, the negative terminal is connected to negative terminal of the battery and positive terminal to positive terminal of battery.

Current and voltage in series arrangement

I. Current in series arrangement

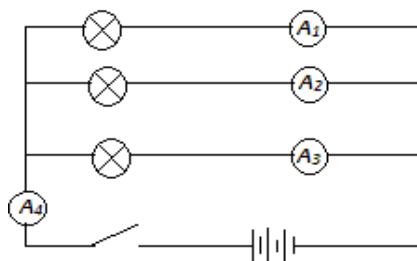
- Consider the set-up below:



Current and Voltage in Parallel Circuit Arrangement

I. Current in Parallel

- Consider the circuit shown below:



- When the switch is closed, it is observed that:

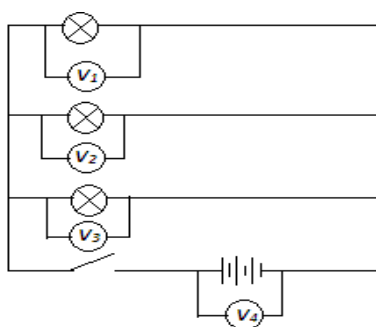
Reading on A_1 + reading on A_2

+ reading on A_3 = Reading on A_4

- Therefore, the sum of the currents in parallel circuits is equal to the total current. The total current flowing into junction is equal to total current flowing out.

II. Voltage in Parallel

- Consider the diagram below:



- When the switch is closed, it is observed that:

Reading on V_1 = reading on V_2 = reading on V_3
= reading on V_4

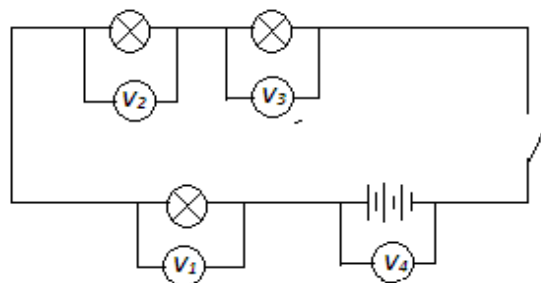
- When the switch is closed, it is observed that;

Reading on A_1 = reading on A_2 = reading on A_3 = reading on A_4

- Therefore, when components are connected in series, same current flows through each of the components even if the components are not identical

II. Voltage in Series Arrangement

- Consider the set-up below



- When the switch is closed, it is observed that:

Reading on V_1 + reading on V_2 + reading on V_3 = reading on V_4

- Therefore, when components are connected in series, the sum of the voltage drop across the components is equal to the voltage supply.

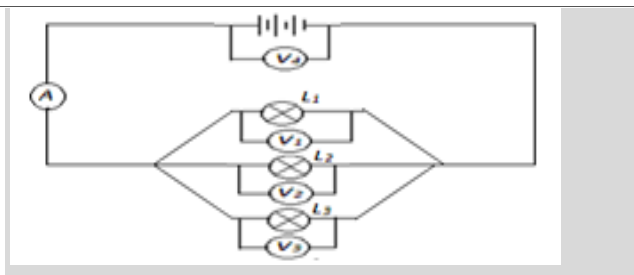
Exercise

- Using the diagram below, find:

I. The current passing through L_1 in the figure below given that ammeter A reads 1.2A and currents through L_2 and L_3 are 0.34A and 0.52A respectively.

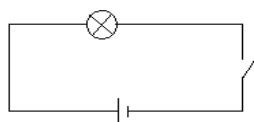
II. V_1 , V_2 and V_3 readings, given that V_4 reads is 3.0V

- **Therefore**, for components in parallel arrangement, the same voltage drops across each of them (since their terminals are at the same electric potential).

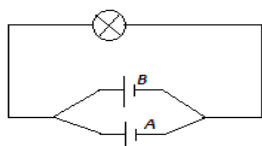


2. In the circuit shown below, what is the p.d across the bulb, and the switch when:

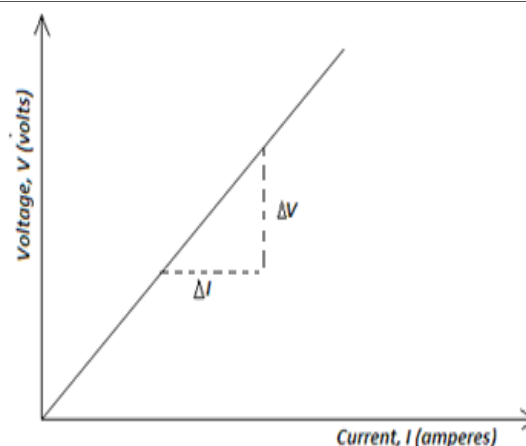
- The switch is open?
- The switch is closed?



3. Define a volt
4. Two cells, A and B connected with parallel are in series with a bulb as shown below.



- Copy the diagram to show where the ammeter should be connected in order to measure the current through cell A
- Voltmeter should be connected to measure the potential difference across both bulb and cell B



- From the graph, voltage is directly proportional to the current and this is graphical representation of Ohm's law.
- The slope of the graph gives **resistance**. SI unit of resistance is **the ohm(Ω)**

$$\frac{\Delta V}{\Delta I} = R$$

- **Multiples of an ohm are:**

$$1 \text{ kilohm } (1 \text{ k}\Omega) = 1000\Omega (10^3\Omega)$$

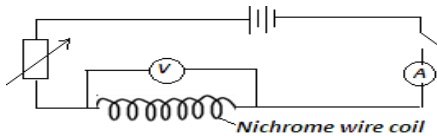
$$1 \text{ megohm } (1 \text{ M}\Omega) = 1000000\Omega (10^6\Omega)$$

Definition of an ohm

- An ohm refers to the resistance of a conductor when a current of 1 A flowing through it causes a voltage

Ohm's Law

- Ohm's law relates the **voltage** across the conductor and the **current** flowing through it. It is after the **physicist George Simon Ohm**. It states that, **"the current flowing through a conductor is directly proportional to the potential difference across its ends provided that temperature and other physical properties are kept constant"**.



- Mathematically, Ohm's law can be expressed as:

$$V \propto I$$

$$V = RI$$

- $\therefore V = IR$, Where V is the potential difference, I is the current and R a constant of proportionality called **resistance**.
- If several values of current and their corresponding values of voltage for nichrome wire are obtained and a graph of voltage against current plotted, a straight line through the origin is obtained.

3. In an experiment to investigate the V - I relationship for a conductor, the following results were obtained

P.d (V)	2.0	3.0	4.0	6.0
Current, I (A)	1.0	1.5	2.0	3.0
Resistance, $R(\Omega)$				

1. Copy and complete the table

drop of 1 V across its ends.

- The reciprocal of resistance is a quantity called **conductance**, $\left(\frac{1}{R}\right)$ whose SI units is Ω^{-1} or **Siemens (S)**

Example

A current of 6mA flows through a conductor of resistance 4k Ω . Calculate the voltage across the conductor.

Solution

$$V = IR$$

$$V = 6 \times 10^{-3} \times 4 \times 10^3 = 24 \text{ V}$$

Exercise

- Calculate the current in mill-amperes flowing through a conductor of conductance 0.2 m Ω^{-1} when a 15V source is connected to it
- In order to start a certain law a current of 36A must flow through the starter motor. Calculate the resistances of the motor given that the battery provides a voltage of 12V ignore the internal resistance of the battery.

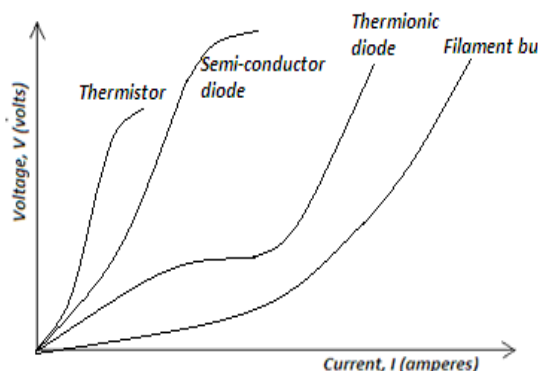
Factors Affecting the Resistance of a Metallic Conductor

1. Temperature

- Resistance of a metallic conductor increases with temperature. This is because heating increases the vibration of atoms thereby increasing the collisions per cross-section area. The opposition to the flow of electrons thus increases as temperatures.

2. Length of the conductor

<p>II. <i>Plot a graph of I against the voltage</i></p> <p>III. <i>Determine the resistance of the conductor</i></p> <p>IV. <i>Comment on the nature of the conductor</i></p> <p>4. <i>Two cells, each of 1.5 V are used to drive a current through a wire AB of resistance 90Ω</i></p> <p>I. <i>Calculate the current in the circuit</i></p> <p>II. <i>What would be the difference, if any to the current, if the two cells are connected in parallel?</i></p>	<ul style="list-style-type: none"> The resistance of a uniform conductor of a given material is directly proportional to its length i.e. $R \propto l \dots\dots (1)$ $R = \text{constant} \times l$ $\frac{R}{l} = \text{constant}$ <p>3. <i>Cross- section area of the conductor</i></p> <ul style="list-style-type: none"> The resistance of a metallic conductor is inversely proportional to its cross-sectional area, A. A conductor with larger cross-sectional area has many free electrons to conductor hence better conductivity. $R \propto \frac{1}{A} \dots\dots\dots (2)$ $R = \text{constant} \times \frac{1}{A}$ $RA = \text{constant}$ <p><i>Resistivity of a metallic conductor</i></p> <ul style="list-style-type: none"> Resistivity is the resistance of sample of material of unit length and unit cross sectional area at a certain temperature. Combining equations (1) and (2) above; $R \propto \frac{l}{A}$ $R = \frac{\rho l}{A}$ $\rho = \frac{RA}{l}, \text{ where } \rho \text{ is the resistivity of the conductor}$ <ul style="list-style-type: none"> The <i>SI unit of resistivity is the ohm meter (Ωm)</i>
<p>Ohmic and non-ohmic conductors</p> <p><i>Ohmic conductors</i></p> <ul style="list-style-type: none"> These are conductors which obey Ohm's law and therefore voltage drop across them is directly proportional to current through them e.g. metal conductors like nichrome and electrolytes like copper sulphate. For ohmic conductors a graph of voltage against current is a straight line through the origin. <p><i>Non- ohmic conductors</i></p> <ul style="list-style-type: none"> These are conductors which do not obey Ohm's law and therefore their resistance changes with current flow e.g. thermistor, thermionic diode, filament bulb, semiconductor diode etc. A graph of voltage against current for non-ohmic conductors is not a straight. 	<p><i>Example</i></p>



Electrical Resistance

- Electrical resistance is the opposition offered by a conductor to the flow of electric current.
- A material with high conductance has very low electrical resistance e.g. copper. Electrical resistance is measured using **an ohmmeter**.

Exercise

1. Given that the resistivity of nichrome is $1.1 \times 10^{-6} \text{ m}$, what length of nichrome wire of a diameter 0.42 m is needed to make a resistor of 20Ω ?

2. Two wires X and B are such that the radius of Y is twice that of X and the length of Y is twice that of X. If the two are of same material, determine the ratio

$$\frac{\text{resistance of X}}{\text{resistance of Y}}$$

Resistors

- These are conductors specially designed to offer particular resistance to the flow of electric current. The symbol of resistor is



Two meters of a resistance wire, area of cross sectional 0.50 mm^2 , has a resistance of 220Ω . Calculate:

- The resistivity of the metal
- The length of the wire which, connected in parallel with the 2 meter length, will give a resistance of 2.00Ω .

Solution

$$l = 2 \text{ m}, \quad A = 0.50 \text{ mm}^2 = 5.0 \times 10^{-5} \text{ m}^2,$$

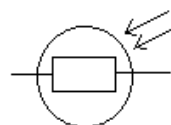
$$R = 220 \Omega$$

$$\rho = \frac{RA}{l}$$

$$\rho = \frac{220 \Omega \times 5.0 \times 10^{-5} \text{ m}^2}{2 \text{ m}} = 5.50 \times 10^{-3} \Omega \text{ m}$$

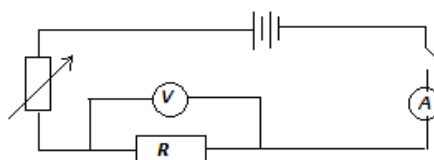
b) Light dependent resistor (LDR)

- Its resistance decreases with increases in light intensity.



Methods of Measuring Resistance

1. Voltmeter-ammeter method



- In this method, current I through the resistor R and corresponding voltage V across it are obtained and

Types of resistors

1. Fixed resistors

- These are resistors designed to give fixed resistance e.g. wire wound resistors, carbon (colour code) resistors etc.

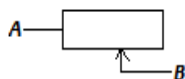
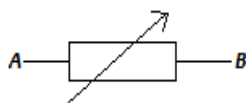


2. Variable resistors

- These are resistors whose resistance can be varied. They include:

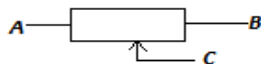
a) Rheostat

- This is two terminal variable resistor represented by any of the symbols below in electrical circuits.



b) Potentiometer

- This is a three terminal variable resistor represented by the symbol below.



3. Nonlinear resistors

- These are resistors in which current flowing through them does not change linearly with the voltage applied. They include:

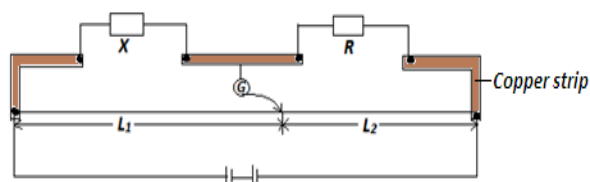
resistance of the resistor is determined using the expression $R = \frac{V}{I}$.

Disadvantages of voltmeter ammeter method

- It is not accurate since voltmeter takes some current and therefore not all current passes through the resistor

2. The meter-bridge method

- In this method a resistor of known resistance is used in the determination of resistance of another resistor whose resistance is not known.
- The figure below shows a set-up of Meter Bridge.



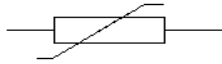
- The bridge is balanced by adjusting the variable resistor L until there is no galvanometer deflection i.e. pointer at zero mark. At balanced state:

$$\frac{X}{R} = \frac{L_1}{L_2}$$

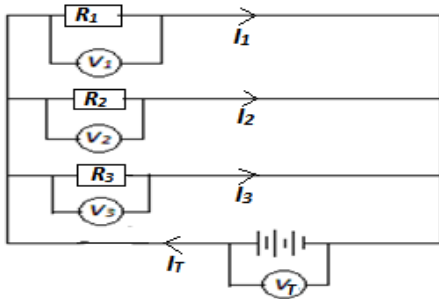
•

a) Thermistor

- This is a temperature dependent resistor whose resistance decreases with increase in temperature. Its electrical symbol is as below.

**Resistors Networks****1. Resistors connected in parallel**

- Consider three resistors R_1 , R_2 and R_3 connected in parallel as shown below:



$$I_T = I_1 + I_2 + I_3$$

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\text{But } V_T = V_1 = V_2 = V_3$$

(for resistors in parallel)

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For n resistors in parallel

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- If two resistors R_1 and R_2 are connected in parallel then the equivalent resistance R_E is given by

$$\frac{1}{R_E} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

a) Calculate the effective resistance**Solution**

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_T} = \frac{1}{4} + \frac{1}{3} + \frac{1}{10} + \frac{1}{4} = 0.9333$$

$$R_T = \frac{1}{0.9333} = 1.074 \Omega$$

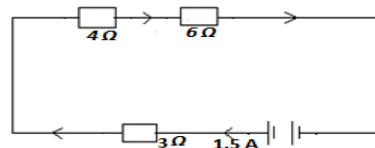
b) The current through the 10Ω resistor**Solution**

Since the resistors are in parallel, the voltage drop across each of them is the same i.e. 4.5 V.

$$I = \frac{V}{R}$$

$$I = \frac{10}{4.5} = 2.222 \text{ A}$$

2. The figure below shows 3 resistors in series connected to power source. A current of 1.5A flow through the circuit.



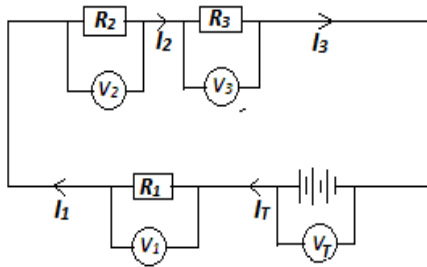
Calculate:

- The total resistance
- The voltage across the source
- The voltage drop across each resistor

Solution

$$R_g = \frac{R_1 R_2}{R_1 + R_2}$$

2. Resistors connected in series



$$V_T = V_1 + V_2 + V_3$$

$$I_T R_T = I_1 R_1 + I_2 R_2 + I_3 R_3$$

$$\text{But } I_T = I_1 = I_2 = I_3$$

$$R_T = R_1 + R_2 + R_3$$

$$\text{a) } R_T = R_1 + R_2 + R_3$$

$$R_T = 4 + 6 + 3 = 13 \, \Omega$$

$$\text{b) } V = IR$$

$$V = 1.5 \times 13 = 19.5 \, \text{V}$$

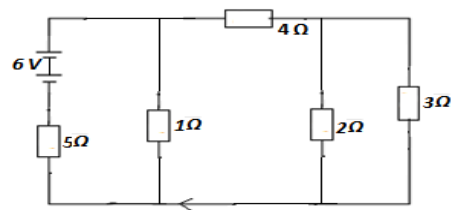
$$\text{c) } V_{4\Omega} = 1.5 \times 4 = 6 \, \text{V}$$

$$V_{6\Omega} = 1.5 \times 6 = 9 \, \text{V}$$

$$V_{3\Omega} = 1.5 \times 3 = 4.5 \, \text{V}$$

Exercise

1. The figure below shows five resistors and a source of voltage of 6V.



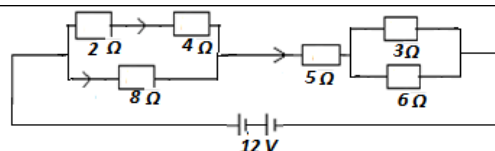
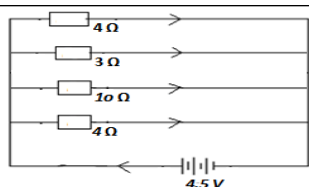
a) Find the effective resistance of the circuit

b) Calculate the current through

3. Six resistors are connected in a circuit as shown in the figure below.

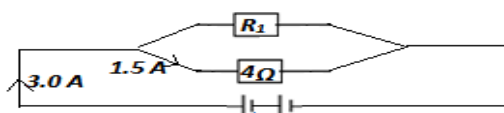
Examples

1. The circuit diagram in the figure below shows 4 resistors connected across a 4.5 V supply



- Calculate the total resistance of the circuit
- The total current in the circuit
- The current through the 3Ω resistor
- The current through the 8Ω resistor

3. Two resistors connected in parallel as shown below

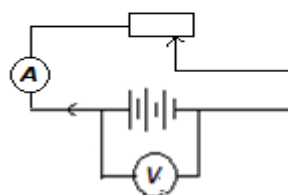


- Calculate:
 - the current that passes through R_1
 - Terminal p.d across the battery

Experimental Determination of Internal Resistance and Emf

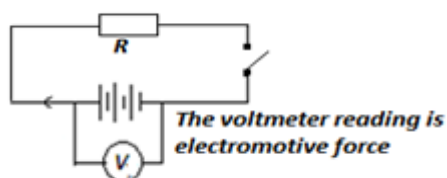
Method 1

- Consider the set up below:

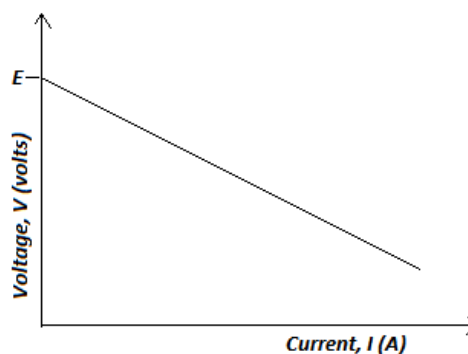


Electromotive Force (Emf) and Internal Resistance r

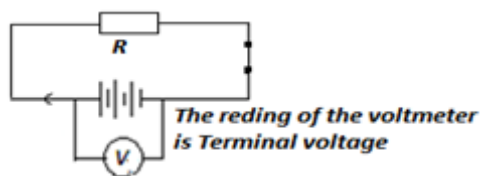
- Electromotive force (E)** of the cell refers to the potential difference across its terminals when no charge is flowing out of it i.e. when the circuit is open.
- Terminal voltage (V)** is the voltage drop across the terminals of the cell or battery when charge is flowing out of it and it is due to external resistance R .



If several values of current and their corresponding values of voltage are collected and graph of voltage V against current I is plotted. It is a straight line of negative slope cutting through the voltage axis when extrapolated.



- Using the equation $E = V + Ir$ and therefore $V = E - Ir$, the **slope** of the graph gives the **internal resistance** of the cell while the **voltage-intercept** gives the **emf (E)** of the cell.



- **Internal resistance (r)** refers to the opposition offered by the source of electromotive force to the flow of the current that it generates.
- **Lost voltage** is the difference between electromotive force and terminal voltage and it is due to internal resistance.

Relationship between Emf (E), Terminal Voltage (V) and Internal Resistance (r)

- Consider a resistor R connected in series with a cell of internal resistance r
- The internal resistance of the cell r is considered to be connected in series with the external resistor, R
- The current flowing through the circuit is given by:

$$\text{current} = \frac{\text{e.m.f}}{\text{total resistance}}$$

$$I = \frac{E}{R + r}$$

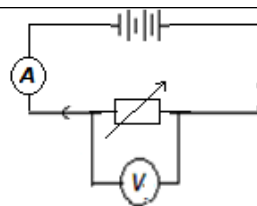
$$E = I(R + r)$$

$$E = IR + Ir \Rightarrow E = V + Ir$$

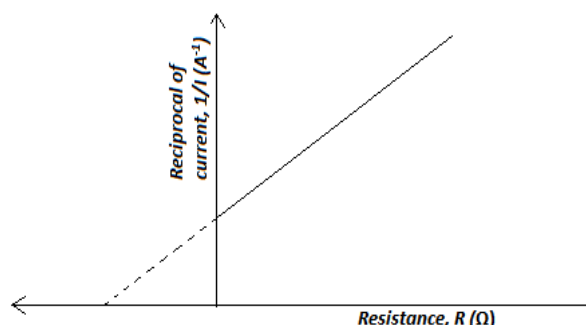
- Where IR = terminal voltage, Ir = lost voltage

- The **gradient** of the graph gives $\frac{1}{E}$ and therefore the electromotive force of the cell can be obtained while the **R -intercept** gives **internal resistance of the cell r** .

Examples



- If several values of current and their corresponding values of voltage are collected and graph of reciprocal of current $\frac{1}{I}$ against R is plotted, a straight line with positive gradient which passes through $\frac{1}{I}$ axis is obtained.



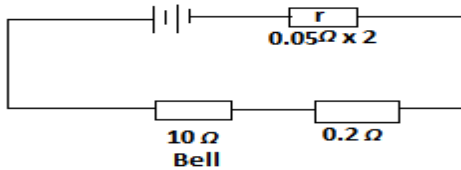
Exercise

1. The table below shows reading obtained in an experiment to determine the e.m.f, E and internal resistance R of a accumulator

External resistance, R (Ω)	0.3 5	0.3 75	2. 75
Current, I (A)	2.5	1.0	0.

1. Two dry cells each of the internal resistance 0.05Ω and connected in series are used to operate an electric bell of resistance 10Ω . The wiring of the circuit has a resistance of 0.2Ω . If the bell requires a current of 0.2A to ring, to what value can the combined emf fall before the bell comes to ring?

Solution

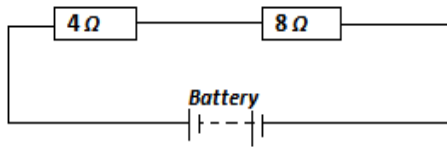


$$E = I(R + r)$$

$$E = 0.2 [(10 + 0.2) + 0.05 \times 2] = 2.06 \text{ V}$$

2. You are provide with resistors of values 4Ω and 8Ω

a) Draw a circuit diagram showing the resistors in series with each other and with battery.



b) Calculate total resistance of the circuit (assume negligible internal resistance)

Solution

$$R_T = R_1 + R_2$$

$$R_T = 4 + 8 = 12 \Omega$$

c) Given that the battery has an emf of 6V and internal resistance of 1.33Ω , calculate the current through

i. 8Ω when the two are in series.

Solution

$$I_{8\Omega} = I_{4\Omega} = \frac{E}{R + r}$$

$$= \frac{6}{12 + 1.33} = 0.4501 \text{ A}$$

ii. 4Ω resistor when the two are in parallel.

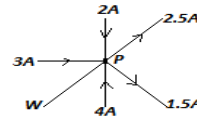
			5
Reciprocal of current, $\frac{1}{I}$			

a) Draw a suitable circuit used to get the above results

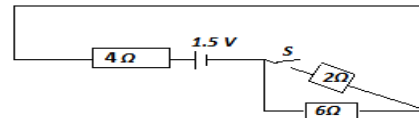
b) Plot the graph of $\frac{1}{I}$ against R

c) Determine the values of internal resistance r and electromotive force

2. The circuit in the figure below shows the current at junction P . Find the amount and direction of the current that passes through the wire W .



3. Three resistors are connected as shown below



Calculate:

a) The total resistance in the circuit when:

i. S_1 is open

ii. S_2 is closed

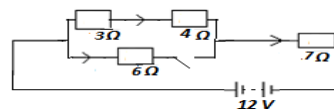
b) The current through each of the resistors when:

i. S_1 is open

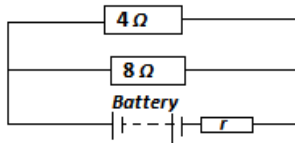
ii. S_1 is closed

c) The potential difference across each resistor when S_1 is closed

4. A battery of emf 12V and internal resistance of 0.6Ω is connected as shown below



a) Calculate the current through the 3Ω resistor when switch is:

Solution

$$R_T = \frac{R_1 R_2}{R_1 + R_2} + r$$

$$R_T = \frac{4 \times 8}{4 + 8} + 1.33 = 4\Omega$$

$$I_{4\Omega} = \frac{V_{4\Omega}}{R_{4\Omega}} = \frac{E - V_r}{R_{4\Omega}}$$

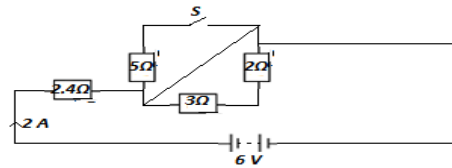
$$I_{4\Omega} = \frac{6 - \frac{6}{4} \times 1.33}{4} = 1.001 \text{ A}$$

i. Open

ii. Closed

b) Find the total potential different across the 7Ω resistors when S is open

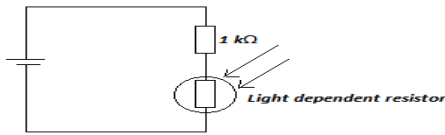
5. A cell of emf of 6.0V and drives current of 2.0A through R_1 when switch S is open



Calculate:

- The current through the 2Ω resistor
- The internal resistance of the cell
- The current through each of the resistors when the switch S is closed

The circuit below can be used as a light sensor.



- Explain how it works as conditions change from pitch darkness to bright light
- If the resistance of LDR in dim light is $1 \times 10^4 \Omega$ calculate the p.d across $1 \text{ k}\Omega$ resistor

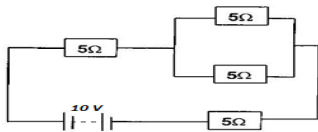
Further Exercise

1. A student learnt that a battery of eight dry cells each 1.5 V has a total emf of 12 V the same as a car battery. He connected in series eight new dry batteries to his car but found that they could not start the engine. Give a reason for this observation

2.a) You are required to determine the resistance per unit length of a nichrome wire x , you are provided with d.c power supply, an ammeter and voltmeter.

- Draw a circuit diagram to show how you would connect the circuit.
- Describe how you would use the circuit in (a) (i) above to determine the resistance per unit length of x .

2. Four 5Ω resistors are connected to a 10 V d. c. supply as shown in the diagram below.

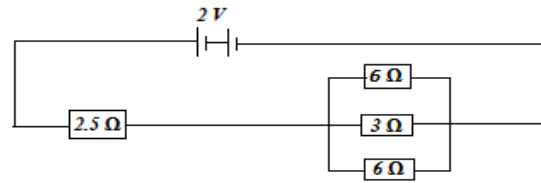


Calculate:-

- The effective resistance in the circuit.
- The current I following in the circuit (through the source)

3. Study the circuit diagram below and determine the potential drop across the 3Ω resistor.

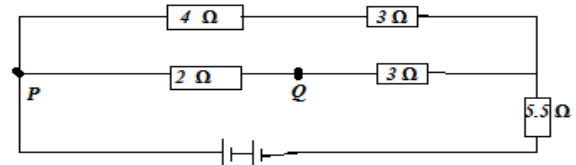
8. a) In the circuit diagram shown, calculate the effective resistance between Y and Z .



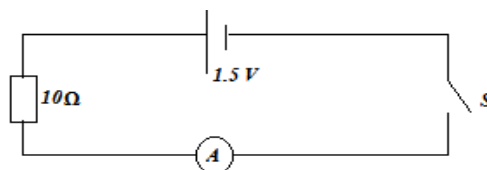
- b) Determine the current through the 3Ω resistor.

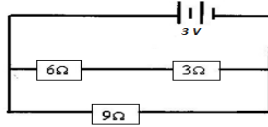
One of the 6Ω resistors has a length of 1 m and cross-sectional area of $5.0 \times 10^{-6} \text{ m}^2$. Calculate the resistivity of the material.

9. In the circuit diagram five resistors are connected to a battery of emf. 4 V , and negligible internal resistance. Determine:



- The total resistance of the circuit.
 - The current flowing through the 5.5Ω resistor.
 - The potentials at points Y and O .
 - The potential difference between Y and O .
11. A student wishes to investigate the relationship between current and voltage for a certain device X . In the space provide, draw a circuit diagram including two cells, rheostat, ammeter, voltmeter and the device X that would be suitable in obtaining the desired results.
12. In the circuit diagram shown in figure 7, the ammeter has negligible resistance. When the switch S is closed, the ammeter reads 0.13 A .





4. *State two conditions that are necessary for a conductor to obey Ohm's law.*
5. *a) Two resistors R_1 and R_2 are connected in series to a 10V battery. The current flowing then is 0.5A. When R_1 only is connected to the battery the current flowing is 0.8A. Calculate the*
 - i. *Value of R_2*
 - ii. *Current flowing when R_1 and R_2 are connected in parallel with the same battery.*
6. *A current of 0.08A passes in circuit for 2.5 minutes. How much charge passes through a point in the circuit?*
7. *An ammeter, a voltmeter and a bulb are connected in a circuit so as to measure the current flowing and the potential difference across both. Sketch a suitable circuit diagram for the arrangement.*

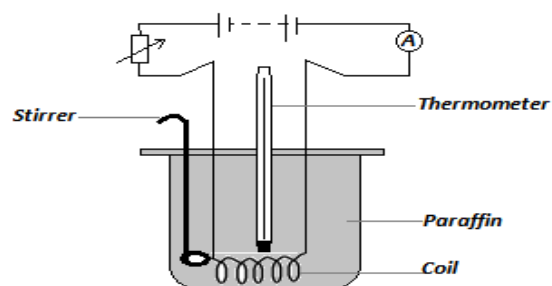
Chapter Eight HEATING EFFECT OF AN ELECTRIC CURRENT	
Specific objectives	Content
<p>By the end of this topic the learner should be able to:</p> <ul style="list-style-type: none"> a) Perform and describe experiments to illustrate heating effect of an electric current b) State the factors affecting heating by electric current c) Derive the equations for electrical energy and electrical power d) Identify devices in which heating effect of an electric current is applied e) Solve numerical problems involving electrical energy and electrical power 	<ul style="list-style-type: none"> 1. Simple experiments on heating effect 2. Factors affecting electrical energy, $W = VIt$, $P = VI$ 3. Heating devices: electrical kettle, electrical iron box, bulb filament, electric heater 4. Problems on electrical energy and electrical power

Introduction

- When an electric current passes through a conductor, it generates heat energy. This is called the **heating effect of an electric current** and it is due to the resistance offered by the conductor to the current. Heating effect of an electric current was first investigated by **James Joule, a Manchester brewer** in UK.

Demonstrating Heating Effect of an Electric Current Using a Coil of Wire

- The set-up below can be used to experimentally demonstrate heating effect of an electric current in the laboratory.



- **Precaution.** the coil should be fully immersed in water but should not touch the bottom or walls of the beaker.
- **Observation:** It is observed that the temperature of water increases with resistance, current and time.
- **Explanation:** Electrical energy is converted to heat energy resulting in a rise in temperature. The heat energy increases with resistance, current and time.

c) Time for which current flows through

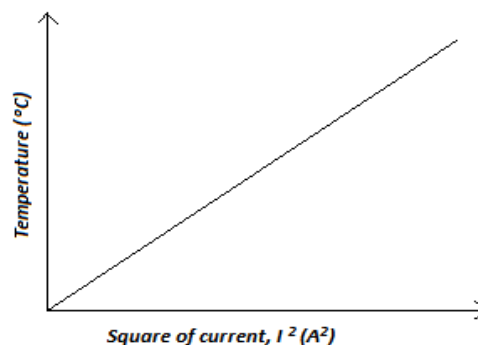
Factors Affecting Heating by Electric Current

The heat produced by a conductor carrying current depends on:

a) Amount current passing through the conductor

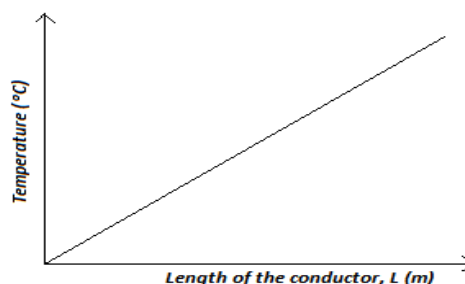
- The heat produced is directly proportional to the square of current through the conductor provided that same conductor is used for the same time. i.e

$$\text{heat energy} \propto I^2.$$



b) Resistance of the conductor

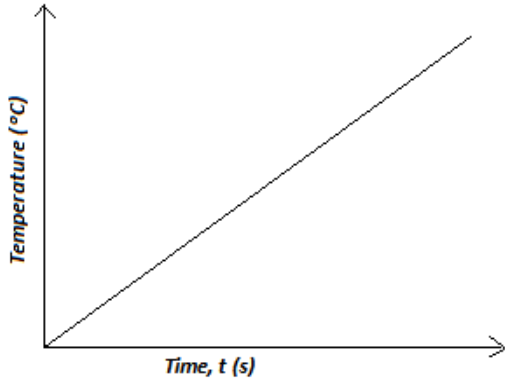
- Heat produced in a conductor carrying current is directly proportional to resistance of the conductor provided current and time are constant.
heat energy $\propto R$.



Examples

conductor

- Keeping current and resistance constant heat produced in a conductor is directly proportional to the time for which current flows.
Heat energy $\propto t$.



V v dA`

Q

Electrical Energy, E

- Consider a current I flowing through a conductor of resistance R for a time t . If a potential difference V drops across the ends of the conductor, then;

$$V = \frac{W}{Q}$$

(from definition of potential difference)

$$E = W = VQ$$

(where W is the electrical work done in moving

1. An electric bulb rated 40W is operating on 240V mains. Determine the resistance of its filament.

Solution

$$P = \frac{V^2}{R}$$

$$40 = \frac{240^2}{R}; \Rightarrow R = 1440 \Omega$$

2. When a current of 2A flows in a resistor for 10 minutes, 15KJ of electrical energy is dissipated. Determine the voltage across the resistor.

Solution

$$E = VIt$$

$$15 \times 1000 = V \times 2 \times 10 \times 60$$

$$V = \frac{15000}{1200} = 12.5 \text{ V}$$

3. How many 100W electric irons could be safely connected to a 240V moving circuit fitted with a 13A fuse?

Solution

$$P = VI$$

$$(\text{No. of irons}) \times 1000 = VI$$

$$\text{No. of irons} = \frac{13 \times 240}{1000}$$

$$= 3.12 = 3 \text{ irons}$$

Exercise

charge Q)

This is electrical work done is converted to heat energy, E

but current, $I = \frac{\text{charge } Q}{\text{time, } t}; \Rightarrow Q = It$

$$E = V(It); \Rightarrow E = VIt$$

Since, $V = IR$ (from ohms law); electrical energy can also be expressed as;

$$E = VIt = (IR)It; \Rightarrow E = I^2RT$$

Electrical Power, P_E

➤ Power is the rate of doing well

$$\text{power} = \frac{\text{work}}{\text{time}}$$

$$= \frac{\text{electrical energy}}{\text{time}} = \frac{VIt}{t};$$

$$\Rightarrow \text{Electrical power, } P_E = VI$$

Since $V = IR$, electrical power can also be expressed as

$$P_E = (IR)I$$

$$P_E = I^2R$$

$$\text{Or from } I = \frac{V}{R}; \quad P = V\left(\frac{V}{R}\right)$$

$$P_E = \frac{V^2}{R}$$

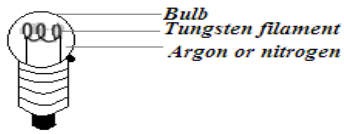
1. A heater of resistance R_1 is rated P watts, V volts while another of resistance R_2 is rated $2P$ watts, $\frac{V}{2}$ volts. Determine $\frac{R_1}{R_2}$
2. State THREE factors which affect heating by an electric current.
3. What is power as it relates to electrical energy?
4. An electrical appliance is rated as 240V, 200W. What does this information mean?
5. An electrical heater is labelled 120W, 240V.

Calculate;

- a. The current through the heating element when the heater is on.
 - b. The resistance of the element used in the heater.
6. An electric toy is rated 100W, 240V. Calculate the resistance of the toy when operating normally.
 7. An electric bulb with a filament of resistance 480Ω is connected to a 240V mains supply. Determine the energy dissipated in 2 minutes.

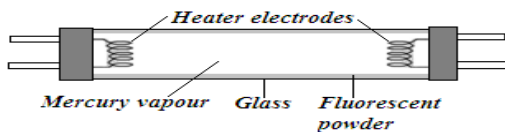
Electrical Devices for Lighting

a) Filament lamps



- When current flows through the filament, it glows white hot and therefore produces light.
- The filament is made of **tungsten** metal due to its very high melting point (3400°C).
- The bulb is filled with **inert gas like argon and nitrogen** to prevent oxidation of the filament.

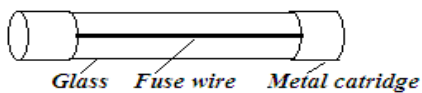
b) Fluorescent lamps



- They are efficient than filament lamps because they last much longer and have low running cost.
- It consists of the **mercury vapor** which produces ultraviolet radiation when the lamp is switched on. The radiation makes the **powder** on the inside of the tube produce **visible light (fluoresce)**.

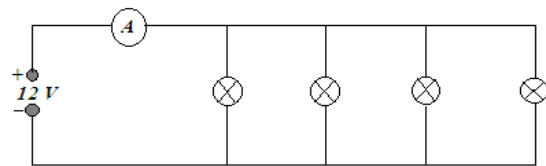
Electrical Devices for Heating

A) Fuse



- A fuse is a short length of wire of material with low **melting point** (tinned

3. A current of 3.3A is passed through a resistor of 25Ω for 2 hours calculate the electrical energy converted to heat energy in 20 minutes.
4. An electric current iron consumes 2.9MJ of energy in 1 hour 10 minutes when converted to the mains power supply of 240V . Calculate the amount throu
- 5.
6. gh the filament in the electric iron
7. In the circuit shown in the figure below each bulb is rated $6\text{V}, 3\text{W}$,



- a) Calculate the current through each bulb, when the bulbs are working normally.
 - b) How many coulombs of charge pass in 6 seconds through each bulb?
 - c) What would the ammeter read when all the bulbs are working normally.
 - d) Calculate the electrical power delivered by the battery.
8. Starting from electrical power, P , generated in a conductor show that $P = \frac{V^2}{R}$, where the symbols their usual meanings.

copper), which melts and breaks the circuit when current through it exceeds a certain value. This ***protects electrical appliances*** and prevents fire outbreaks.

Other electrical heating devices include:

- a) Radiant electric heater*
- b) Electrical iron box*
- c) Electric kettle*
- d) Hot wire ammeter*
- a)***

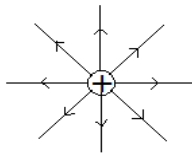
Revision Exercise

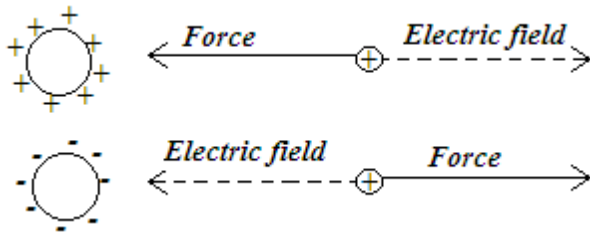
- 1. A touch bulb is called 2.5v, 0.4A. What is the power rating of the bulb?***
- 2. An electric bulb is labeled 50w, 240v, calculate***
 - b) The resistance of the filament used in the lamp***

c) *The current through the filament when the bulb works normally*

Chapter Nine

ELECTROSTATICS II

Specific objectives	Content
<p>By the end of this topic the learner, should be able to:</p> <ol style="list-style-type: none"> Sketch electric field patterns around charged bodies. Describe charge distribution on conductors of various shapes. Define capacitance and state its SI unit. Describe charging and discharging of a capacitor (calculation involving curves not required). State the factors affecting the capacitance of a parallel plate capacitor State the applications of capacitors. Solve numerical problems involving capacitors. 	<ol style="list-style-type: none"> Electric field patterns. Charge distribution on conductors. Spherical and pear shaped conductors. Action at points; lighting arrestors. Capacitance, unit of capacitance (farad, microfarad), factors affecting capacitance. Applications of capacitors. Problems on capacitors (using $Q = CV, C_T = C_1 + C_2, \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$)
<p>Electric Field</p> <p>❖ An electric field refers to the region where a charged body experiences a force of attraction or repulsion.</p>	 <p>2. Isolated negative point charge</p> <p><i>The field lines are radially inwards towards the negative charge</i></p>
<p>Direction of an Electric Field</p> <p>❖ The direction of an electric field at a particular point is defined as the direction in which a unit positive charge is free to move when placed at that point.</p>	



Electric Line of Force (Electric Field Line)

This is the ***path*** along which a unit positive charge would tend to move in the electric field.

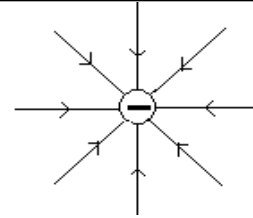
Properties of Electric Field Lines

1. Electric field lines start at 90° from the positive charge and end on the negative charge at 90° .
2. They do not cross each other.
3. They tend to contract or expand so that they never intersect each other

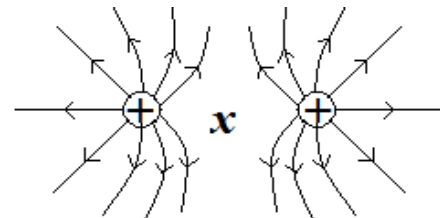
Electric Field Patterns

Isolated positive point charge

The field lines are radially outwards from the positive charge

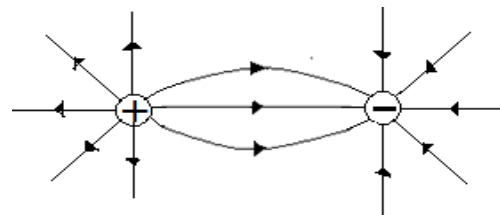


3. Two equal positive point charge

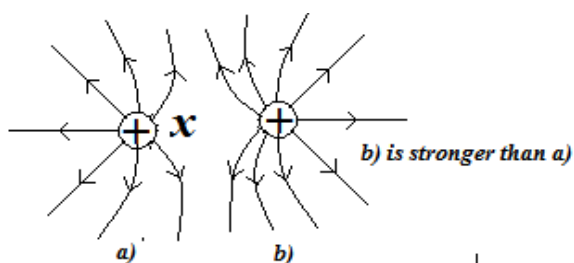


x is the neutral region. The resultant electric field in the region is zero

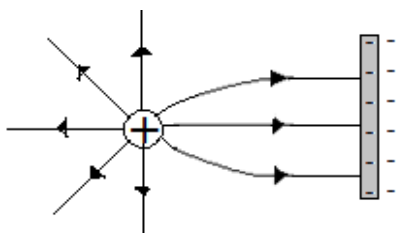
4. Two equal unlike point charge



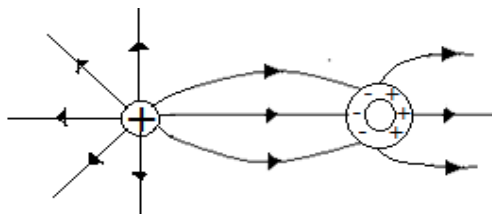
5. Two unequal positive point charge



6. Positive point charge and a straight metal plate having negative charge

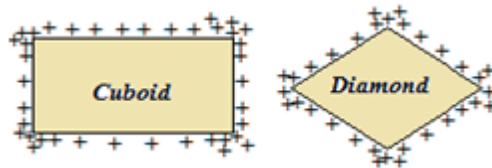
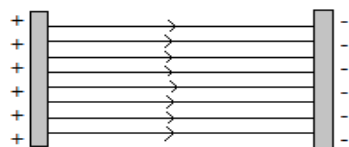


7. Positive point charge and uncharged ring placed in the electric field



8. Two parallel metal plates having opposite charge and placed close together.

The electric field between them is uniform i.e. field lines equally placed.



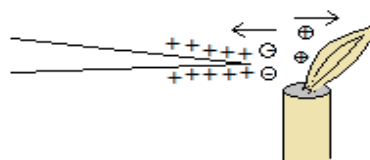
- ❖ No charge is found on the outside of a ***hollow conductor***. For a hollow conductor, the charge resides on the outside.

Point Action

- ❖ ***Point action*** refers to the fast loss or gain of charge at sharp points due to high charge concentration at the points.

Demonstration of Point Action

- ❖ A highly charged sharp point is placed close to a Bunsen burner flame.
- ❖ It is ***observed*** that the flame is blown away.

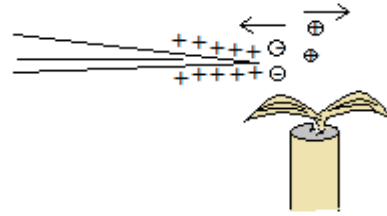
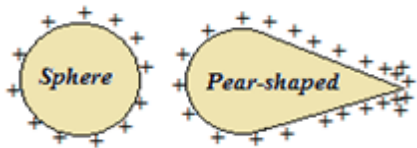


Explanation

- ❖ Burning flame contains positive and negative ions. When the sharp [point is brought close to the flame, negative ions are attracted to the sharp point, while positive ions are repelled away from the sharp point. As the positive ions are repelled, they create an ***"electric wind"*** which blows.
- ❖ If the conductor is brought very close to the flame, the flame splits.

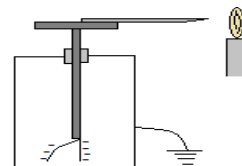
Charge Distribution on Conductors

- ❖ **Distribution** of charge on the surface of a conductor *depends on the shape of the conductor*.
- ❖ For **spherical conductor**, charge is **uniformly distributed** on the surface.
- ❖ For **pear shaped conductor**, charge is concentrated at **the sharp point**.
- ❖ For **Cuboid and diamond conductors** high charge density is at **the vertices**.



Example

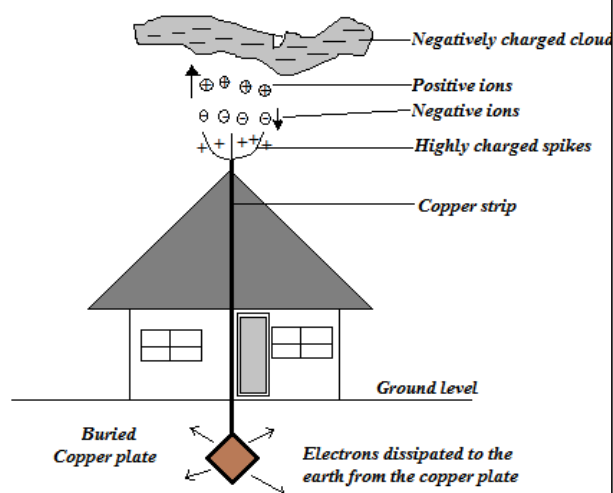
1. A candle flame is placed near a sharp pointed pin connected to the cap of a negatively charged electroscope as shown below.



	<p>State and explain what is observed on the leaf of the electroscope.</p>
<p><u>Solution</u></p> <p><i>It is observed that the leaf collapses. This is because the sharp point ionizes the air around it. The negative ions (electrons) move to the sharp end to ionize the positive charged air ions which are attracted to the cap (or positive air ions neutralize the negative charge on the sharp point).</i></p> <p>2. It is dangerous to carry a pointed umbrella when it is raining. Explain.</p> <p><u>Solution</u></p> <p><i>The sharp point of the umbrella attracts charge readily to neutralize the charge in the cloud which may electrocute the person holding the umbrella.</i></p>	<div data-bbox="1062 516 1318 730" data-label="Image"> </div> <ol style="list-style-type: none"> 1. Is electrical field strength a scalar or vector quantity? Explain 2. Explain how negatively charged pointed edge gets discharged by itself. 3. It is not advisable to take shelter under a tree when it is raining. Explain. 4. What is the purpose of the spikes on the lightning arrestor?

Application of Point Action

Point action is applied in the working of the **lighting arrestor**.



Working Mechanism of the Lightning Arrestor

- ❖ When a negatively charged cloud passes over the arrestor it induces positive charge on the spikes and negative charge on the plate.
- ❖ The negative charge on the plate is immediately discharged to the surrounding ground.
- ❖ Negative ions are attracted to the spikes and are discharged by giving up their electrons.
- ❖ At the same time, positive ions are repelled upwards from the spikes and they neutralize the negative charge on the clouds.

Exercise

1. The fig. shows a hollow negatively charged sphere with metal disk attached to an insulator placed inside. State what would happen to the leaf of an uncharged electroscope if the metal disk were brought near the cap of electroscope. Give a reason for your answer.

Capacitors

- ❖ A capacitor is a device used for storing charge.

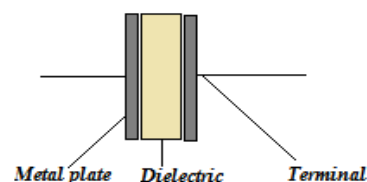
- ❖ Capacitor symbol is

Types of Capacitors

1. Paper capacitors
2. Electrolytic capacitors
3. Variable air capacitor
4. Parallel plate capacitor

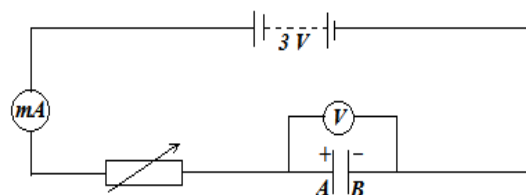
Parallel Plate Capacitor

- ❖ A parallel plate capacitor consists of two metal plates separated by an insulating material called **dielectric**.



Charging a Capacitor

- ❖ The circuit diagram below shows a set-up for charging a capacitor.

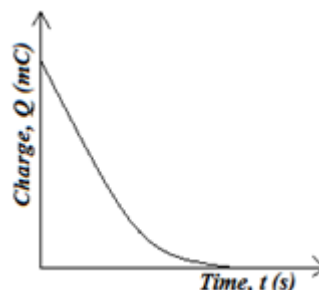
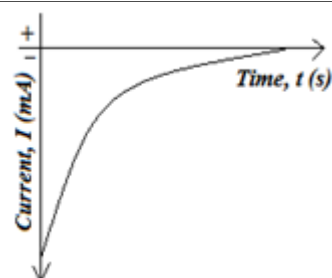
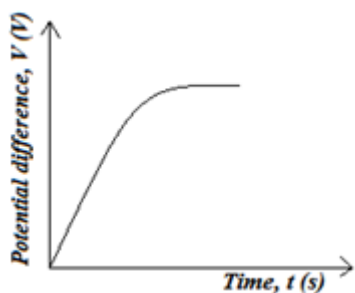
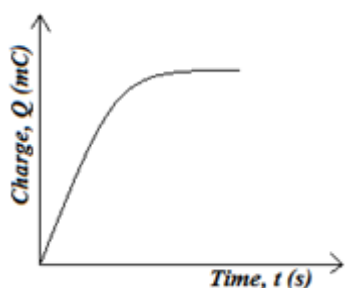
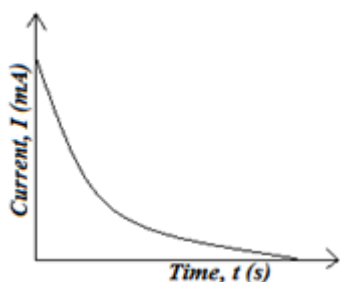


- ❖ **Observation:** When the switch is closed it is **observed** that milli-ammeter reading decreases while voltmeter reading increases.

- ❖ **Explanation:** Negative charge flow from the negative

terminal of the battery to plate **B** of the capacitor. At the same rate negative charge flow from the plate **A** of capacitor to positive terminal of battery. Therefore, equal positive and negative charges appear on the plates and oppose the flow of electrons which causes.

- ❖ The charging current drops to zero when the capacitor is fully charged.
- ❖ Potential difference across the capacitor also develops during charging.



Capacitance

- ❖ Capacitance is defined as the charge stored in a capacitor per unit voltage.

$$\text{capacitance, } c = \frac{\text{charge, } Q}{\text{voltage, } V}$$

$$c = \frac{Q}{V}$$

- ❖ The *SI unit* of capacitance is *the farad (F)*

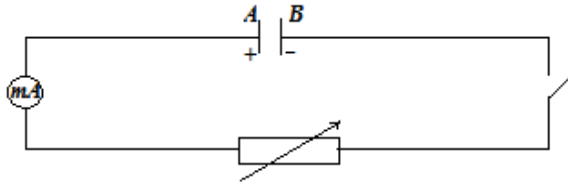
Definition of the farad

- ❖ A farad is the capacitance of a body if a charge of 1 coulomb raises its potential by 1 volt

Submultiples of the farad

Discharging a Charged Capacitor

- ❖ The circuit for discharging is as below.



- ❖ **Observation:** It is observed that milliammeter reading decreases from maximum value to minimum. The pointer deflects in opposite direction to that during charging.
- ❖ **Explanation:** During discharging charge flow in the opposite direction i.e. from plate to A until the charge on the plates is zero.
- ❖ Potential difference across the capacitor practically diminishes to zero.

1 microfarad ($1\mu F$) = $10^{-6} F$

1 Nano farad ($1\mu f$) = $10^{-9} F$

1 Pico farad ($1pF$) = $10^{-12} F$

Factors Affecting the Capacitance of a Parallel- Plate Capacitor

1. Area of Overlap of the Plates, A

- ❖ Capacitance is directly proportional to the area of overlap of the plates ($C \propto A$)
- ❖ If the positive plate is connected to the cap of a positively charged electroscope, the divergence of the leaf increases as the area of overlap increases

2. Distance of Separation of the Plates, d

- ❖ Capacitance is inversely proportional to distance of separation of the plates ($C \propto \frac{1}{d}$)
- ❖ If the positive plate is connected to the cap of a positively charged electroscope, the divergence of the leaf decreases as the distance of the separation increases.

3. Nature of the Dielectric

- ❖ **Dielectric** refers to the insulating material between the plates of a parallel plate capacitor.
- ❖ Permittivity ϵ of the insulating material is a constant dependent on the medium between the plates.
- ❖ If the plates are in a vacuum, the constant is denoted by ϵ_0 (epsilon naught) and its value is $8.85 \times 10^{-12} \text{ Fm}^{-1}$

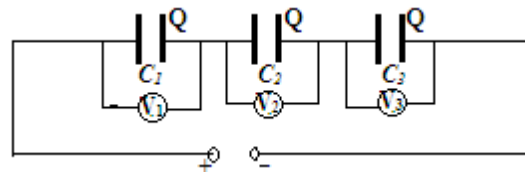
Expression for capacitance

$$C \propto \frac{A}{d}$$

Capacitor Networks

a) Capacitors in Series

- ❖ Consider the capacitors arrangement below.



Charge on C_1 = change on C_2 = change on C_3 = Q

but $V = V_1 + V_2 + V_3$ (I.e. components in series)

$$\text{and } V = \frac{Q}{C}$$

$$C = \frac{\epsilon A}{d}, \text{ where } \epsilon = \text{permittivity}$$

Examples

1. Two plates of a parallel plate capacitor are 1mm apart and each has an area of 6cm^2 . Given that the potential difference between the plates is 90V, calculate the charge stored in the capacitor. (Take $\epsilon_0 = 8.85 \times 10^{-12}\text{Fm}^{-1}$)

Solution

$$C = \frac{\epsilon A}{d}$$

$$C = \frac{8.85 \times 10^{-12}\text{Fm}^{-1} \times 6 \times 10^{-4}\text{m}^2}{1 \times 10^{-3}} = 5.31 \times 10^{-12}\text{F}$$

$$Q = CV$$

$$Q = 5.31 \times 10^{-12} \times 90 = 4.779 \times 10^{-10}\text{C}$$

2. Find the separation distance between two plate if the capacitance between them is $6 \times 10^{-12}\text{F}$ and the enclosed area is 3.0cm^2 (take $\epsilon_0 = 8.85 \times 10^{-12}\text{Fm}^{-1}$)

Solution

$$C = \frac{\epsilon A}{d}$$

$$6 \times 10^{-12}\text{F} = \frac{8.85 \times 10^{-12}\text{Fm}^{-1} \times 3 \times 10^{-4}\text{m}^2}{d}$$

$$d = \frac{8.85 \times 10^{-12}\text{Fm}^{-1} \times 3 \times 10^{-4}\text{m}^2}{6 \times 10^{-12}\text{F}} = 4.425 \times 10^{-4}\text{m}^2$$

Exercise

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

where C_T is the combined capacitance

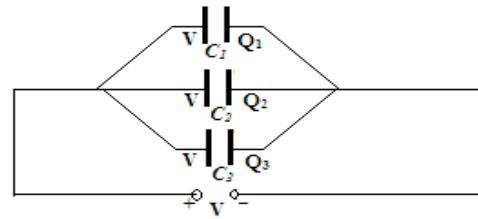
- ❖ For only two capacitors, C_1 and C_2 are in series,

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$\text{Therefore } C_T = \frac{C_1 C_2}{C_1 + C_2}$$

B) Capacitors in Parallel

- ❖ Consider the arrangement below.



- ❖ The p.d across each of the capacitors is the same as the p.d across the source since they are connected in parallel.

$$Q_T = Q_1 + Q_2 + Q_3$$

$$\text{but } Q = CV$$

$$C_T V = C_1 V_1 + C_2 V_2 + C_3 V_3$$

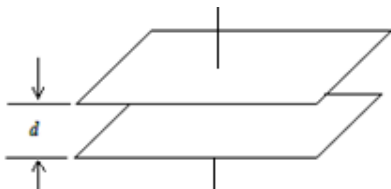
$$C_T = C_1 + C_2 + C_3$$

- ❖ **Note:** Treat charge as current since current is the rate of flow of charge.

Examples

1. A charge of $4 \times 10^4 \text{ C}$ was stored in a parallel plate capacitor when a potential difference of 5 V was applied across the capacitor. Work out the capacitance of the capacitor.

2. The figure below represents two parallel plates of a capacitor separated by a distance d . Each plate has an area of A square units. Suggest two adjustments that can be made so as to reduce the effective capacitance.



Find:

a) The combined capacitance

Solution

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_T} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$$

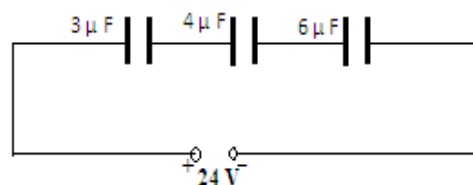
$$\frac{1}{C_T} = \frac{4 + 3 + 2}{12} = 0.75$$

$$C_T = \frac{1}{0.75} = 1.333 \mu\text{F}$$

b) The total charge

Solution

1. Three capacitors of capacitance $3 \mu\text{F}$, $4 \mu\text{F}$ and $6 \mu\text{F}$ are connected to a potential difference of 24 V as shown below.



$$Q_{3\mu\text{F}} = C_{3\mu\text{F}} V_{3\mu\text{F}} ;$$

$$Q_{3\mu\text{F}} = 3 \mu\text{F} \times 9.0048 \text{ V} = 27.0144 \mu\text{C}$$

$$Q_{4\mu\text{F}} = C_{4\mu\text{F}} V_{4\mu\text{F}} ;$$

$$Q_{4\mu\text{F}} = 4 \mu\text{F} \times 9.0048 \text{ V} = 36.0192 \mu\text{C}$$

$$Q_{2\mu\text{F}} = C_{2\mu\text{F}} V_{2\mu\text{F}} ;$$

$$Q_{2\mu\text{F}} = 2 \mu\text{F} \times 9.0048 \text{ V} = 18.0096 \mu\text{C}$$

$$Q_{5\mu\text{F}} = C_{5\mu\text{F}} V_{5\mu\text{F}} ;$$

$$Q_{5\mu\text{F}} = 5 \mu\text{F} \times 14.9952 \text{ V} = 74.976 \mu\text{C}$$

Exercise

$$Q = CV$$

$$Q = 1.333\mu\text{F} \times 24\text{ V} = 32\text{ }\mu\text{C}$$

c) *The charge on each capacitor*

Solution

Since the capacitors are in series, charge on each of them is the same and is equal to total charge i.e. $32\text{ }\mu\text{C}$

d) *The voltage across the $4\mu\text{F}$ capacitor*

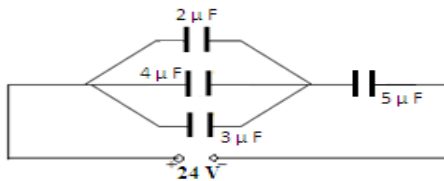
Solution

$$V = \frac{Q}{C}$$

$$V = \frac{32\text{ }\mu\text{C}}{4\text{ }\mu\text{F}} = 8\text{ V}$$

2. Four capacitors of capacitance $3\mu\text{F}$, $4\mu\text{F}$, $2\mu\text{F}$, and $5\mu\text{F}$ are arranged as shown below.

Find:



a) *The combined capacitance*

Solution

Capacitors $3\mu\text{F}$, $4\mu\text{F}$ and $2\mu\text{F}$ are in parallel and their total capacitance is in series with the $5\mu\text{F}$ capacitor.

$$\frac{1}{C_T} = \frac{1}{C_1 + C_2 + C_3} + \frac{1}{C_4}$$

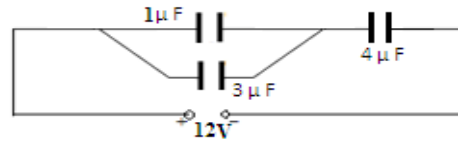
$$\frac{1}{C_T} = \frac{1}{2 + 4 + 3} + \frac{1}{5} = \frac{14}{45}$$

$$C_T = \frac{45}{14} = 3.214\text{ }\mu\text{F}$$

b) *The total charge*

Solution

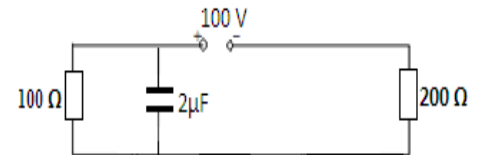
1. In the circuit below $C_1 = 4\mu\text{F}$, $C_2 = 3\mu\text{F}$ and $C_3 = 1\mu\text{F}$. Given that $V = 12\text{ V}$, calculate:



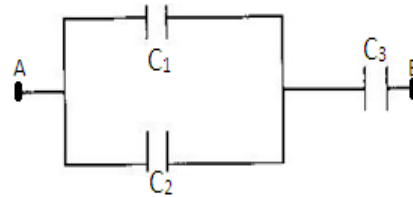
I. *The charge on each capacitor*

II. *The voltage across each capacitor*

2. In the circuit shown below calculate the charge on the capacitor

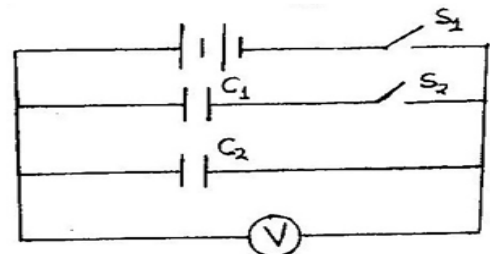


3. The figure below Shows part of a circuit containing three capacitors. Write an expression for C_T . (The effective capacitance between A and B.)



4. *State the law of electrostatic charge.*

5. *The capacitors in the circuit in Figure below are identical and initially uncharged.*



6. *Switch S_1 is opened and switch S_2 closed. Determine the final reading of the voltmeter, V .*

7. *In the circuit diagram shown in figure below each cell has*

$$Q_T = C_T V_T$$

$$Q_T = 3.214 \mu\text{F} \times 24 \text{ V} = 74.976 \mu\text{C} \text{ or } 74.976 \times 10^{-6} \text{ C}$$

c) The charge on each capacitor

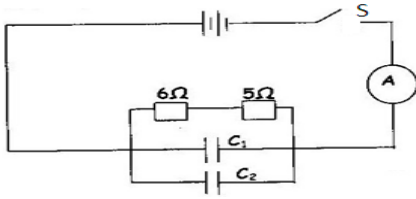
Solution

Voltage across $3\mu\text{F}$, $4\mu\text{F}$ and $2\mu\text{F}$ is the same since they are in parallel;

$$V_T - V_{5\mu\text{F}} = V_T - \frac{Q_T}{C_{5\mu\text{F}}}$$

$$24 - \frac{74.976 \mu\text{C}}{5\mu\text{F}} = 9.0048 \text{ V}$$

an emf of 1.5 and internal resistance of 0.5Ω . The capacitance of each capacitor is $1.4\mu\text{F}$.



8. When the switch s is closed determine the:

- Ammeter reading
- Charge on each capacitor

9. Three capacitors of $1.5\mu F$, $2.0\mu F$ and $3.0\mu F$ are connected in series to p.d. of $12V$. Find:-

- The combined capacitance.
- The total charge stored in the arrangement.
- The charge in each capacitor.

10. In the circuit of the figure 3 $C_1=2\mu F$, $C_2=C_3=0.5\mu F$ and E is a $6V$ battery. Calculate the total charge and p.d across C_1

Energy Stored in a Capacitor

- ❖ Charging a capacitor involves doing work against repulsion of the negative plate to more electrons flowing in and attraction of the positive plate on electrons flowing out.
- ❖ This work is stored in form of potential energy. The energy may be converted to heat, light or other forms.

work done (Energy stored in capacitor)
= average charge \times potential difference

$$\text{Energy stored} = \frac{1}{2}QV$$

$$\text{Energy stored} = \frac{1}{2}CV^2 \text{ (since } Q = CV\text{)}$$

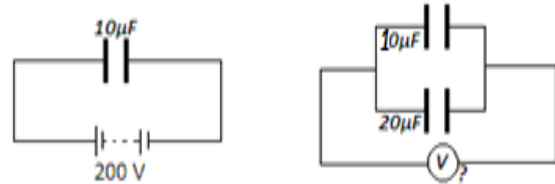
$$\text{Energy stored} = \frac{Q^2}{2C} \text{ (since } V = \frac{Q}{C}\text{)}$$

- ❖ A graph of charge, Q against voltage, V is a straight line through the origin.

2. A $10\mu F$ capacitor is charged to a p.d of $200V$ and isolated. It is then connected in parallel to a $20\mu F$ capacitor. Find:

- The resultant potential difference.

Solution



$$Q_T = Q_{10\mu F} + Q_{20\mu F}$$

$$10 \times 10^{-6} F \times 200 V = (10 \times 10^{-6})V + (20 \times 10^{-6})V$$

$$V = \frac{10 \times 10^{-6} F \times 200 V}{(10 \times 10^{-6})F + (20 \times 10^{-6})F} = 10 V$$

- The energy stored before connection.

Solution

$$\text{Energy stored} = \frac{1}{2}CV^2$$

$$\text{Energy stored} = \frac{1}{2} \times 10 \times 10^{-6} F \times (200 V)^2 = 0.2 J$$

- The energy in the two capacitors after connection.

Solution

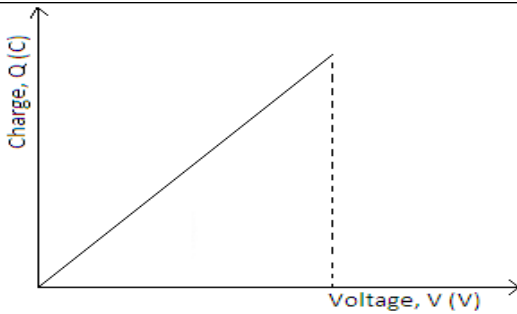
$$\text{Energy stored} = \frac{1}{2}CV^2; \text{ Energy stored} = \frac{1}{2}(C_1 + C_2)V^2$$

$$\text{Energy stored} = \frac{1}{2}(10 \times 10^{-6} + 20 \times 10^{-6})10^2 = 1.5 \times 10^{-3} J$$

- The energy difference between II and III above and comment on your answer.

Solution

$$(0.2 - 0.0015) J = 0.1985 J$$



The energy is converted to heat and light

Examples

1. A $4\mu\text{F}$ capacitor is charged to a potential difference of 80V . Find the energy stored in it

Solution

$$\text{Energy stored} = \frac{1}{2}CV^2$$

$$\text{Energy stored} = \frac{1}{2}(4 \times 10^{-6}\text{F}) \times 80\text{V}$$

$$\text{Energy stored} = 1.6 \times 10^{-4} \text{ J}$$

Exercise

1. A $10\mu\text{F}$ capacitor is charged by an 80V supply and then connected across an uncharged $20\mu\text{F}$ capacitor. Calculate:

I. The final p.d across each capacitor.

II. The final charge on each.

The initial and final energy stored by the capacitors.

2. A $2\mu\text{F}$ capacitor is charged to a potential of 200V , the supply is disconnected. The capacitor is then connected to another uncharged capacitor. The p.d. across the parallel arrangement is 80V . Find the capacitance of the second

capacitor.

3. A $5\mu\text{F}$ capacitor is charged to a p.d of 200v and isolated. It is then connected to another uncharged capacitor of $10\mu\text{F}$. Calculate:

- I. The resultant p.d
- II. The charge in each capacitor.

4. In an experiment to study the variation of charge stored on capacitor and the potential difference across it, the following results were obtained.

Charge Q (μC)	0.0	0.1	0.2	0.3	0.4	0.56
	8	6	4	2	0	
p.d (v)	2.0	4.0	6.0	8.0	10.0	14.0

5. Plot a graph of charge Q . against p.d

6. Use your graph to determine:-

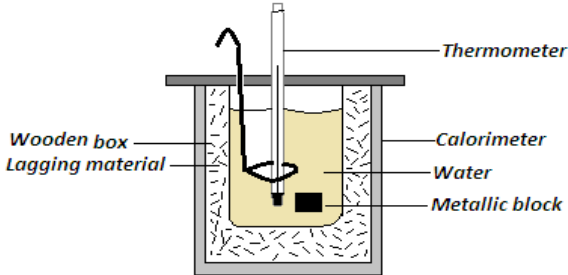
a. Capacitance of the capacitor.

Energy stored in the capacitor when the p.d across its plate is 10V.

Applications of Capacitors

1. *Used in smoothening circuits to smoothen the d.c output in rectification process*
2. *Used in reduction of sparking in induction coil contact. Variable capacitor is used in tuning circuit of a radio receiver in which it is connected in parallel to inductor*
3. *Capacitors are used in delay circuits designed to give intermittent flow of current in car indicators.*
4. *A capacitor is included in flash circuit of a camera in which it discharges instantly to flash.*

<u>Specific objectives</u>	<u>Content</u>				
<p>By the end of this topic, the leaner should be able to:</p> <p>a) Define heat capacity and specific heat capacity</p> <p>b) Determine experimentally specific heat capacity of solids and liquids</p> <p>c) Define specific latent heat of fusion and specific latent heat of vaporization</p> <p>d) Determine experimentally the specific latent heat of fusion of ice and the specific latent heat of vaporization of steam.</p> <p>e) State the factors affecting melting point and boiling point</p> <p>f) Explain the functioning of a pressure cooker and a refrigerator</p> <p>g) Solve problems involving quality of heat.</p>	<p>1. Heat capacity, specific heat capacity units (experimental treatment required)</p> <p>2. Latent heat of fusion, latent heat of vaporization, units (experimental treatment necessary)</p> <p>3. Boiling and melting</p> <p>4. Pressure cooker, refrigerator</p> <p>5. Problems on quantity of heat</p> <p>(Q = MCΔθ, Q = ML)</p>				
<p>Definition of Heat</p> <p>❖ Heat is form of energy that flows from one body to another due to temperature difference between them.</p> <p>Differences between Heat and Temperature</p> <table><tr><td>Heat</td><td>Temperature</td></tr><tr><td>Heat is a form of energy that flows from one body to another due to temperature difference</td><td>Temperature is the degree of hotness or coldness of a body measured on same scale</td></tr></table>	Heat	Temperature	Heat is a form of energy that flows from one body to another due to temperature difference	Temperature is the degree of hotness or coldness of a body measured on same scale	<p>Solution</p> <p>Q = CΔθ</p> <p>Q = 600JK⁻¹ × (60⁰ - 20⁰)</p> <p>= 24000 J</p> <p>Exercise</p> <p>An electrical heater rated 240 V, 3 A raises the temperature of liquid X from 25° C to 55° C in 10 minutes. Calculate the heat capacity of liquid X.</p>
Heat	Temperature				
Heat is a form of energy that flows from one body to another due to temperature difference	Temperature is the degree of hotness or coldness of a body measured on same scale				

Measured in <i>joules</i>	Measured in <i>kelvin</i>	Specific Heat Capacity, <i>c</i>
Heat Capacity, <i>C</i> <ul style="list-style-type: none"> ❖ Heat capacity refers to the quantity of heat energy required to raise the temperature of a given mass of a substance by one kelvin. $\text{Heat capacity, } C = \frac{\text{heat energy absorbed, } Q}{\text{temperature change, } \Delta\theta}$ $C = \frac{Q}{\Delta\theta} \text{ or } Q = C\Delta\theta$ <ul style="list-style-type: none"> ❖ The <i>SI unit</i> of heat is the joule per kelvin (J K^{-1}) ❖ Note: Different materials have different rates of heat absorption and therefore different heat capacities. 		<ul style="list-style-type: none"> ❖ Specific heat capacity is the quantity of heat required to raise the temperature of a unit mass of a substance by one kelvin (<i>K</i>). $\text{Specific heat capacity, } c = \frac{\text{heat energy absorbed, } Q}{\text{mass, } m \times \text{temperature change, } \Delta\theta}$ $c = \frac{Q}{m\Delta\theta}$ $Q = mc\Delta\theta$ <ul style="list-style-type: none"> ❖ The <i>SI unit</i> of specific heat capacity is the joule per kilogram per kelvin ($\text{J kg}^{-1} \text{K}^{-1}$) ❖ Specific heat capacity can also be expressed as: $\text{Specific heat capacity, } c = \frac{\text{heat capacity, } C}{\text{mass}}$ $\text{Heat capacity, } C = \text{specific heat capacity, } c \times \text{mass, } m$
<p align="center"><u>Examples</u></p> <p>Calculate the quantity of heat required to raise the temperature of a metal block with a heat capacity of 600 J K^{-1} from 20°C to 60°C.</p>		
<ul style="list-style-type: none"> ❖ Note: If two substances of the same mass are subjected to the same amount of heat, they acquire different temperature changes because they have different specific heat capacities e.g. the specific heat capacity of iron is $460 \text{ J kg}^{-1} \text{K}^{-1}$. This means that 1kg of iron would take in or give out 460J of heat when its temperature changes by 1k. 		<p>i. State the precautions that need to be taken to minimize heat losses to the surroundings</p> <p><u>Solution</u></p> <ol style="list-style-type: none"> The calorimeter should be highly polished. The calorimeter should be heavily lagged. The calorimeter should be closed using an insulating lid (lid made of a poor conductor).
<p align="center"><u>Example</u></p> <p>A block of copper of mass 10.0 kg and specific heat capacity $460 \text{ J kg}^{-1} \text{K}^{-1}$ cools from 80°C to 40°C. Find the quantity of heat given out.</p> <p><u>Solution</u></p> $Q = mc\Delta\theta$ $Q = 10 \text{ kg} \times 460 \text{ J kg}^{-1} \text{K}^{-1} (80 - 40) ^\circ \text{C}$ $Q = 184000 \text{ J} = 184 \text{ kJ}$		 <p align="center">Determining specific heat capacity of liquids using method of mixtures</p>

Exercise

1. A block of metal of mass 1.5kg which is suitably insulated is heated from 30°C to 50°C in 8 minutes and 20 seconds by an electric heater coil rated 54watts. Find:

- The quantity of heat supplied by the heater
- The heat capacity of the block
- The specific heat capacity

2 Find the final temperature of water if a heater source rated 50W heats 100g water from 25°C in 5 minutes (specific heat capacity of water is 4200Jkg⁻¹ K⁻¹)

Methods of Determining Specific Heat Capacities**1. Method of Mixtures**

- In this method, a relatively hot substance is mixed with a relatively cold substance. Heat energy is transferred from hot body to cold body until **thermal equilibrium** is established

Determining Specific Heat Capacity of a Solid Using Method of Mixtures**Example**

A lagged copper calorimeter of mass 0.50kg contains 0.4kg of water at 25°C. A metallic solid of mass 1.2kg is transferred from an oven at 350°C to the calorimeter and a steady temperature of 50°C is reached by the water after stirring.

- Calculate the specific heat capacity of the material of the solid (Specific heat capacity of copper is 400Jkg⁻¹ K⁻¹ and that of water 4200Jkg⁻¹ K⁻¹)

Solution

Heat lost by the solid
= heat gained by the water + heat gained by calorimeter

Example

A block of metal of mass 0.15kg at 100°C was transferred to a copper calorimeter of mass 0.4kg containing a liquid of mass 0.8kg at 20°C. The block and the calorimeter with its contents eventually reached a common temperature of 40°C. Given the specific heat capacity of aluminium 900Jkg⁻¹ K⁻¹, and that of copper 400Jkg⁻¹ K⁻¹, calculate the specific heat capacity of the liquid.

Solution

Heat lost by the metal block
= heat gained by liquid + heat gained by calorimeter

$$m_B c_B \Delta\theta_B = m_L c_L \Delta\theta_L + m_C c_C \Delta\theta_C$$

$$0.15 \times 900 \times (100 - 40) = 0.8 \times c_L \times (40 - 20) + 0.4 \times 400 \times (40 - 20)$$

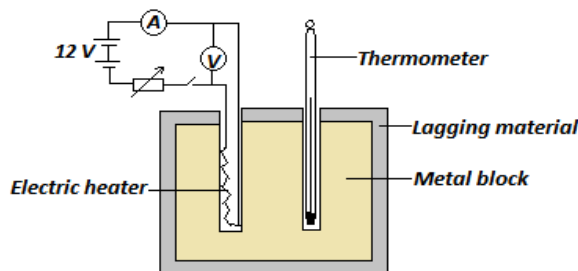
$$c_L = \frac{8100 - 3200}{16} = 306.25 \text{ Jkg}^{-1} \text{ K}^{-1}$$

2. Electrical method

- In this method, electric heating coil supplies the heat energy which is absorbed by other substances.

Determining Specific Heat Capacity of a Metal Block Using Electrical Method

- The set-up that can be used in this case is as shown below.



$$m_s c_s \Delta \theta_s = m_w c_w \Delta \theta_w + m_c c_c \Delta \theta_c$$

$$1.2 \times c_s \times (350 - 50) = 0.4 \times 4200 \times (50 - 25) + 0.5 \times 400 \times (50 - 25)$$

$$c_s = \frac{42000 + 5000}{360} = 130.56 \text{ Jkg}^{-1}\text{K}^{-1}$$

Precautions

- I. The metal block must be highly polished and heavily lagged.
- II. The two holes should be filled with a high oil to improve thermal contact with the heater and thermometer.

Example

A metal block of mass 0.5 kg is heated electrically. If the voltmeter reads 20 V, the ammeter 4A and the temperature of the block rises from 25°C to 95°C in 8 minutes, calculate the specific heat capacity of the metal block.

Solution

Heat supplied by the heater
= heat absorbed by the metal block

$$VIt = m_b c_b \Delta \theta$$

$$20 \times 4 \times (8 \times 60) \text{ J} = 0.5 \text{ kg} \times c_b \times (95 - 25) \text{ K}^{-1}$$

$$c_b = \frac{38400}{35} = 1097.14 \text{ Jkg}^{-1}\text{K}^{-1}$$

Determining Specific Heat Capacity of Liquid Using Electrical Method

Example

In an experiment to determine the specific heat capacity of water an electrical heater was used. If the voltmeter reading was 24 V and that of ammeter 2.0 A, calculate the specific heat capacity of water if the temperature of a mass of 1.5kg of water in a 0.4kg copper calorimeter rose by 6°C after 13.5 minutes.

Change of State

- ❖ Change of a substance from solid to liquid, from liquid to gas or the reverse involves **change of state**.

Latent Heat

- ❖ **Latent heat** refers to amount of heat required to change state of a substance without change in temperature
- ❖ It is the heat energy absorbed or given out during change of state.

Latent Heat of Fusion

- ❖ **Latent heat of fusion** refers to the amount heat required to change the state of a substance from solid to liquid without temperature change.
- ❖ **Note:** When a liquid changes to solid state, latent heat of fusion is given out.

Specific Latent Heat of Fusion, l_f

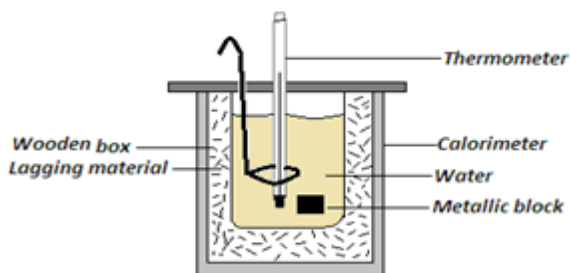
- ❖ **Specific latent heat fusion** refers to the quantity of heat required to change a unit mass of the substance from solid to liquid without change in temperature.

$$l_f = \frac{Q}{m}, \Rightarrow Q = ml_f$$

- ❖ The **SI unit** of specific latent heat of fusion is **the joule per kilogram (Jkg^{-1})**.

Latent Heat of Vaporization

- ❖ **Latent heat of vaporization** refers to the heat required to change the state of a substance from liquid to gas without change in temperature.



Solution

Heat supplied by the heater
= heat gained by water + heat gained by calorimeter

$$VIt = m_w c_w \Delta\theta_w + m_c c_c \Delta\theta_c$$

$$24 \times 2 \times (13.5 \times 60) = 1.5 \times c_w \times 6 + 0.4 \times 400 \times 6$$

$$c_w = \frac{38880 - 960}{9} = 4213.33 \text{ Jkg}^{-1}\text{K}^{-1}$$

Exercise

A man wanted to have a warm bath at 35°C . He had 4.0kg of water in a basin at 90°C . What mass of cold water at 23°C must he have added to the hot water to obtain his choice of bath. Neglect heat losses and take specific heat capacity of water as $4200\text{Jkg}^{-1}\text{K}^{-1}$

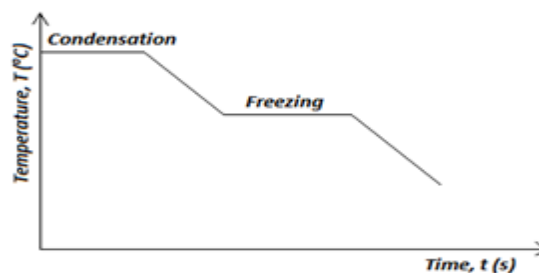
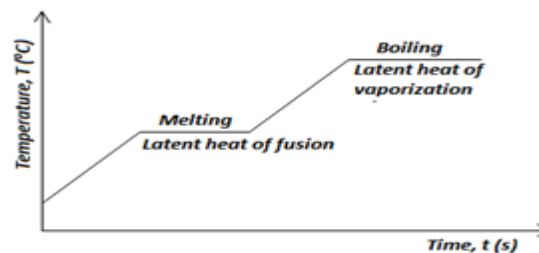
Specific latent heat of vaporization,

- ❖ **Specific latent heat of vaporization** is the quantity of heat required to change a unit mass of a substance from liquid to vapor without change in temperature.

The SI
unit of

specific latent heat of vaporization is **the joule per kilogram (Jkg^{-1})**.

$$l_v = \frac{Q}{m}, \Rightarrow Q = ml_v$$



Examples

1. Calculate the amount of heat required to convert 4kg of ice at -10°C to liquid at 5°C (specific heat capacity of water is $4200\text{Jkg}^{-1}\text{K}^{-1}$, specific heat capacity of ice $=2100\text{Jkg}^{-1}\text{K}^{-1}$, specific latent heat of fusion of ice $=340,000\text{Jkg}^{-1}\text{K}^{-1}$)

Solution

$$Q = m_{\text{ice}}c_{\text{ice}}\Delta\theta_{\text{ice}} + m_{\text{ice}}l_{\text{f ice}} + m_{\text{water}}c_{\text{water}}\Delta\theta_{\text{water}}$$

$$Q = 4 \times 2100 \times (10 - 0) + 4 \times 340000 + 4 \times 4200 \times (5 - 0)$$

$$Q = 84000 + 1360000 + 84000$$

$$Q = 1528000 \text{ J or } 1.528 \text{ MJ}$$

2. A kettle rated at 4.0kW containing 2.0kg of water is left switched on. How long will it take the water to boil dry in the kettle if the initial temperature of water is 20°C (specific heat capacity of water is $4200\text{Jkg}^{-1}\text{K}^{-1}$ specific latent heat of vaporization of water is $2.26 \times 10^6 \text{ Jkg}^{-1}$).

Solution

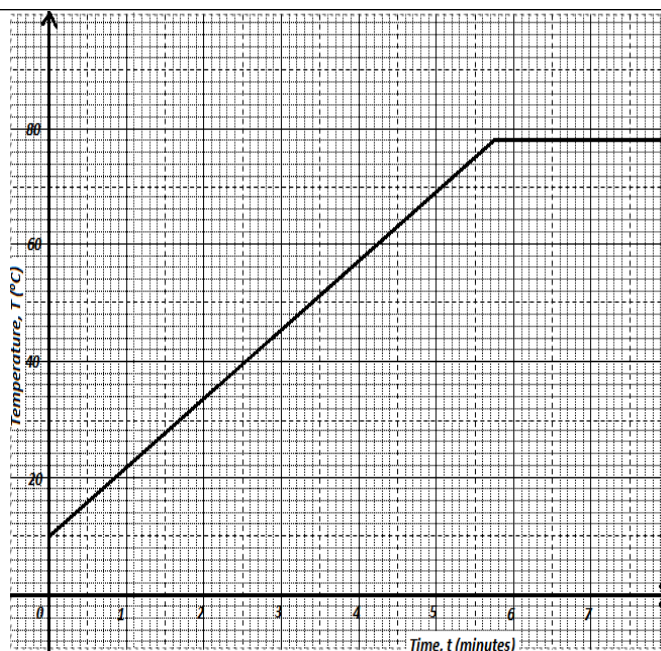
$$Q = Pt = m_{\text{water}}c_{\text{water}}\Delta\theta_{\text{water}} + ml_{\text{V water}}$$

$$4000 \times t = 2 \times 4200 \times (100 - 20) + 2 \times 2.26 \times 10^6$$

$$t = \frac{672000 + 4520000}{4000} = 1298\text{s}$$

Exercise

- Calculate the amount of thermal energy required to change 5g of ice at -10°C to steam at 100°C (specific heat capacity of ice $2.10\text{Jg}^{-1}\text{K}^{-1}$, specific latent heat of fusion of ice 336Jg^{-1} , specific latent heat of vaporization of steam 2260Jg^{-1} , specific heat capacity of water $4.2\text{Jg}^{-1}\text{K}^{-1}$)
- A copper calorimeter of mass 60g contains 100g of oil at 20°C . a piece of ice of mass 28g at 10°C is added to the oil. What mass of ice will be left when the temperature of the calorimeter and its contents will be 10°C ? Specific heat capacity of copper $=0.4\text{Jg}^{-1}\text{K}^{-1}$, specific heat capacity of oil $2.4\text{Jg}^{-1}\text{K}^{-1}$, specific latent heat of fusion of ice 336Jg^{-1})



- From the graph, determine the boiling point of the liquid.
- Determine the heat given out by the heater between the times $t=0.5$ minutes and $t=5.0$ minutes.
 - From the graph determine the temperature change between the times $t=0.5$ minutes and $t=5.0$ minutes.
 - Hence determine the specific heat capacity of the liquid
- 1.8g of vapor was collected from the liquid between the times $t=6.8$ minutes and $t=7.3$ minutes. Determine the specific latent heat of vaporization of the liquid.

Factors affecting melting and boiling points

They are two:

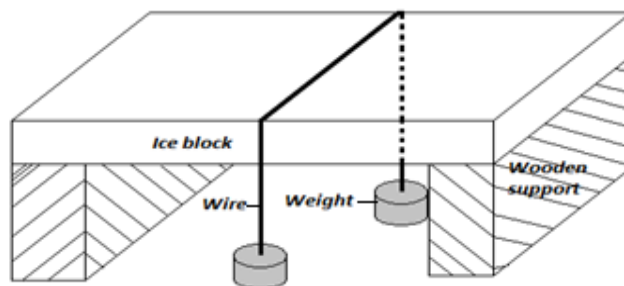
- Pressure
- Impurities

Effect of pressure on melting point

- ❖ Increase in pressure lowers the melting point of a substance.
- ❖ Consider the set-up below.

3. Dry steam is passed into a well lagged aluminium calorimeter of mass 400 g containing 1.2 kg of ice at 0°C . The mixture is well stirred and steam supply cut off when the temperature of the calorimeter and its contents reaches 53°C . Neglecting heat losses, determine the specific latent heat of vaporization of water if 400 g of steam is found to have condensed to water. (specific latent heat of fusion of ice 336Jg^{-1} , specific heat capacity of water $4.2\text{Jg}^{-1}\text{K}^{-1}$, specific heat capacity of aluminium is $900\text{Jkg}^{-1}\text{K}^{-1}$.)

4. a) State two factors that affect the boiling point of a liquid. b) 100g of a liquid at a temperature of 10°C is poured into a well lagged calorimeter. An electric heater rated 50W is used to heat the liquid. The graph in the figure below shows the variation of the temperature of the liquid with time.



❖ It is **observed** that the wire cuts its way through the ice block, but leaves it as one piece.

Explanation

- ❖ The wire exerts pressure on the ice beneath it and therefore makes it melt at a temperature lower than its melting point.
- ❖ The water formed by melted ice flows over the wire and immediately solidifies and gives out latent heat of fusion to copper wire which it uses to melt ice below it.
- ❖ **Note:** Copper wire is used in the experiment because:
 - I. It is a good conductor of heat.
 - II. It has higher thermal conductivity than other metals.

Applications of effects of pressure on melting point of ice

- I. It is applied on ice skating in which weight of the skater acts on ice through thin blades of skates. This melts the ice to form a film of water on which the skater slides.
- II. It can be used in joining two ice cubes under pressure.

Evaporation

- ❖ Evaporation is the process by which a liquid changes to a gas. It occurs at all temperatures (i.e. has no fixed temperature)

Factors affecting rate of evaporation

1. Temperature

Rate of evaporation increases with temperature since increase in temperature increases kinetic energy of molecules and therefore surface molecules easily escape.

2. Surface area

- ❖ Rate of evaporation increases with surface area because many molecules are exposed when surface area is large. Water in basin (a) evaporates faster than the one in (b) in the figure below.



3. Draught

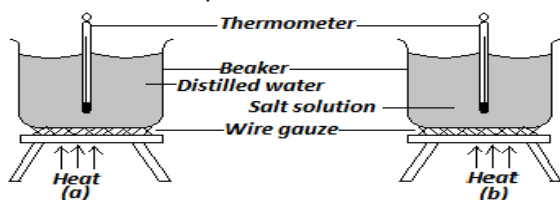
- ❖ Draught increases rate of evaporation since it sweeps away evaporating molecules clearing a way

Effect of impurities on melting point

- ❖ Impurities lower the melting points of substances.
- ❖ This is applied in humid areas during winter in which salt is spread on roads and paths to prevent freezing.

Effect of impurities on boiling point

- ❖ Impurities raise the boiling points of substances. This is why salted food cooks faster than unsalted one.
- ❖ Consider the set-up below.



- ❖ When the liquids are heated to boiling, the boiling point of salt solution is observed to be higher than that of distilled water. This is because impurities raise the boiling point of a liquid.

Effect of Pressure on Boiling Point

- ❖ Decrease in pressure lowers the boiling point of a liquid while increase in pressure raises the boiling point.
- ❖ This is why food takes longer to cook at high altitudes than at low altitudes because pressure at high altitude is higher.
- ❖ It is also the reason as to why water in a sufuria closed with a lid boils faster than the closed one. The steam pressure in a closed sufuria is higher and this raises the boiling point of water making it to boil faster.

❖

for more molecules to escape. This is why clothes dry faster on a windy day.

4. Humidity

- ❖ Increase in humidity lowers rate of evaporation. This is why clothes take long to dry on a humid day.

Effects of Evaporation

- One feels cold when methylated spirit is applied on his head after shaving. This is because the evaporating spirit gets latent heat of vaporization from his body.
- Thin layer of frost forms around the outside of a test tube walls when air is blown through methylated spirit in the tube. This is because the evaporating spirit obtains latent heat of vaporization from walls of the tube creating a cooling effect.

Difference between Evaporation and Boiling

Evaporation	Boiling
It takes place at all temperatures.	It takes place at fixed temperature.
It takes place on the liquid surface only.	It takes place throughout the liquid.
Decreasing atmospheric pressure increases the rate of evaporation.	Decreasing atmospheric pressure lowers the boiling point.

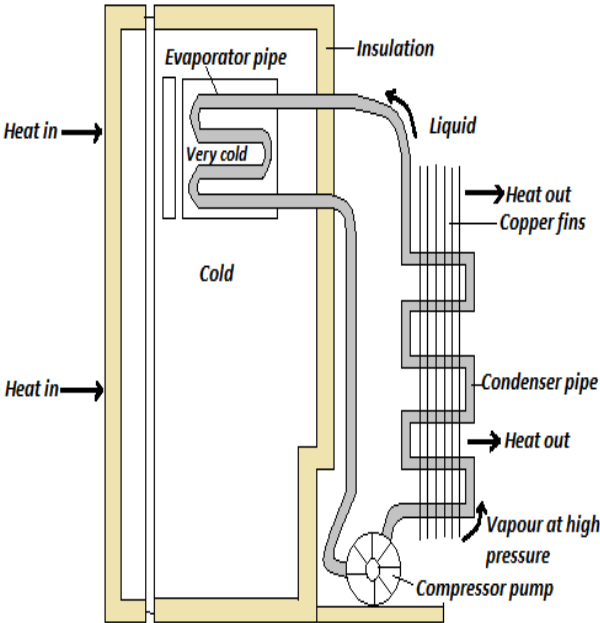
Applications of Cooling by Evaporation

1. Sweating

- ❖ Evaporating sweat absorbs latent heat of vaporization from body and therefore creating a cooling effect.

2. Cooling of Water in a Porous Pot

- ❖ Water seeping out of the pot through pores

	<p>evaporates from surface of pot, creating a cooling effect.</p>
<p>1. The Refrigerator</p>  <ul style="list-style-type: none"> ❖ It consists of a highly volatile liquid (Freon) which takes latent heat of vaporization from contents (food) in refrigerator and evaporates. ❖ The pump removes the vapor into lower coil outside the cabinet where it is compressed and changed to liquid form. ❖ The evaporator and condenser pipes are highly coiled to increase surface area for absorption and loss of heat respectively. ❖ The pipes are made of good conductor of heat (copper) to increase heat conductivity. ❖ The copper fins enhance heat loss to the surrounding at the condenser pipe. 	<p>4. State two factors that would raise the boiling point of water to above 100°C</p> <p>5. a) State what is meant by the term specific latent heat of vaporization</p> <p>b) In an experiment to determine the specific latent heat of vaporization of water, steam at 100°C was passed into water contained in a well-lagged copper calorimeter. The following measurements were made:</p> <ul style="list-style-type: none"> • Mass of calorimeter = 50g • Initial mass of water = 70g • Final mass of calorimeter + water + condensed steam = 123g • Final temperature of mixture = 30°C <p>(Specific heat capacity of water = $4200\text{ J kg}^{-1}\text{ K}$ and specific heat capacity for copper = $390\text{ J kg}^{-1}\text{ K}^{-1}$) Determine the:</p> <ol style="list-style-type: none"> I. Mass of condensed steam II. Heat gained by the calorimeter and water III. Given that L is the specific latent heat of evaporation of steam <ol style="list-style-type: none"> i. Write an expression for the heat given out by steam ii. Determine the value of L. <p>6. A heating element rated 2.5 KW is used to raise the temperature of 3.0 kg of water through 50°C. Calculate the time required to affect this. (Specific heat capacity of water is 4200 J/kg/K)</p> <p>7. State two factors that affect the melting point of ice.</p> <p>8. Steam of mass 3.0g at 100°C is passes into water of mass 400g at 10°C. The final temperature of the mixture is T. The</p>

Revision Exercise

1. An electric heater rated 6000W is used to heat 1kg of ice initially at -10°C until all the mass turns to steam. Given that latent heat of fusion of ice = 334kJ kg^{-1} , specific heat capacity of ice = $2,260\text{J kg}^{-1}\text{ K}^{-1}$, specific heat capacity of water = $4,200\text{J kg}^{-1}\text{ K}^{-1}$ and latent heat of vaporization of water = $2,260\text{kJ kg}^{-1}\text{ K}^{-1}$, calculate the minimum time required for this activity.

2 a) Explain why a burn from the steam of boiling water is more severe than that of water itself?

b) An energy saving stove when burning steadily has an efficiency of 60%. The stove melts 0.03kg of ice at 0°C in 180 seconds. Calculate; -

- I. The power rating of the stove.
- II. The heat energy wasted by the stove.

3 An immersion heater rated 90W is placed in a liquid of mass 2kg. When the heater is switched on for 15 minutes, the temperature of the liquid rises from 20°C to 30°C . Determine the specific heat capacity of the liquid.

container absorbs negligible heat. (Specific latent heat of vaporization of steam = 2260 kJ/kg , specific heat capacity of water = $4200\text{J kg}^{-1}\text{ K}^{-1}$)

- I. Derive an expression for the heat lost by the steam as it condenses to water at temperature T .
- II. Derive an expression for the heat gained by the water.
- III. Determine the value of T .

9. A can together with stirrer of total heat capacity 60J/K contains 200g of water at 10°C . Dry steam at 100°C is passed in while the water is stirred until the whole reaches a temperature of 30°C . Calculate the mass of steam condensed.

10. An immersion heater which takes a current of 3A from 240V mains raised the temperature of 10kg of water 30°C to 50°C . How long did it take?

11. 100g of boiling water are poured into a metal vessel weighing 800g at a temperature of 20°C if the final temperature is 50°C . What is the specific heat capacity of the metal? (Specific Heat capacity of water $4.2 \times 10^3\text{ J/kg/K}$)

12. 0.02kg of ice and 0.01kg of water 0°C are in a container. Steam at 100°C is passed in until all the ice is just melted. How much water is now in the container?

13. In a domestic oil-fired boiler, 0.5kg of water flows through the boiler every second. The water enters the boiler at a temperature of 30°C and leaves at a temperature of 70°C , re-entering the boilers after flowing around the radiators at 30°C . $3.0 \times 10^7\text{ J}$ of heat is given to the water by each kilogram of oil burnt. The specific heat capacity of water is $4200\text{J kg}^{-1}\text{ K}^{-1}$.

- I. Use the information above to calculate the energy absorbed by the water every second as it passes through the boiler.
- II. Use the same information above to calculate the mass of oil which would need to be burnt in order to provide this energy.

14. You are provided with two beakers. The first beaker contains hot water at 70°C . The second beaker contains cold water at 20°C . The mass of hot water is thrice that of cold water. The contents of both beakers are mixed. What is the temperature of the

mixture?

15. Calculate the heat evolved when 100g of copper are cooled from 90°C to 10°C . (Specific Heat Capacity of Copper = 390J/Kg/K).

16. An immersion heater rated 150W is placed in a liquid of mass 5 kg. When the heater is switched on for 25 minutes, the temperature of the liquid rises from 20°C to 27°C . Determine the specific heat capacity of the liquid. (Assume no heat losses)